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FUZZME: A NEW SOFTWARE FOR MULTIPLE-CRITERIA FUZZY EVALUATION

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Abstract. This paper is focused on an introduction of a new software product, which is called FuzzME. This software was developed as a tool for creating fuzzy models of multiple-criteria evaluation and decision making. The type of evaluations employed in the fuzzy models fully corresponds with the paradigm of the fuzzy set theory; the evaluations express the (fuzzy) degrees of fulfillment of corresponding goals. The FuzzME software works with both quantitative and qualitative criteria. The basic structure of evaluation is described by a goals tree. Within the goals tree, aggregation of partial fuzzy evaluations is done either by one of fuzzified aggregation operators or by a fuzzy expert system. The FuzzME software takes advantage of linguistic fuzzy modeling to the maximum extent.
This paper also contains a short summary of other available software product for fuzzy multiple-criteria evaluation.
In this paper, the possibilities of FuzzME are demonstrated on a sample problem - evaluation of a new employee.

1. Introduction

There are many situations which require use of multiple-criteria evaluation models. Such models can be utilized e.q. for evaluation of universities, rating of clients of a bank or for evaluation of new employees. In the chapter 4, the last situation will be used as an example and its solution with FuzzME software will be described more in detail.

In the evaluation models, some of the input data are set expertly (e.g. evaluations of alternatives according to qualitative criteria, partial evaluating functions for quantitative criteria, a choice of a suitable type of aggregation, criteria weights, or eventually, a rule base describing the relation between criteria values and the overall evaluation). Because uncertainty is the typical feature of any expert information, the fuzzy set theory is a suitable mathematical tool for creating such models. For the practical use of the fuzzy models of multiple-criteria

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evaluation, their user-friendly software implementation is necessary. But a good theoretical basis of the used models is crucial, too. The clear and well-elaborated theory of multiple-criteria fuzzy evaluation makes it possible to create an understandable methodics for the software user. And a good methodics is essential for correct application of any software to solving real problems.

There is a large number of papers and books dealing with the theory and methods of multiple-criteria evaluation that make use of the fuzzy approach (e.g. [1], [2], [3], [4]).

The most commonly used software for multiple criteria evaluation and decision making based on fuzzy models is FuzzyTECH [5] even if it was not its main purpose (its main application area is fuzzy control). FuzzyTECH is a general software product that makes it possible to create and use fuzzy expert systems. It also includes neural networks algorithms for deriving fuzzy rule bases from data. Interesting applications of this software to evaluation and decision making in the area of business and finance were published in [6].

In 2000 a Czech software company, TESCO SW Inc., developed a software product whose name is NEFRIT. It uses fuzzy methods for multiple criteria evaluation and decision making. The fuzzy model of evaluation applied there is described in detail in [7] and in the book [8]. The demo version of this software is enclosed to the book [8]. NEFRIT can work with expert fuzzy evaluations of alternatives according to qualitative criteria. The values of quantitative criteria can be either crisp or fuzzy. Evaluating functions for quantitative criteria represent membership functions of partial fuzzy goals. For aggregation, the method of weighted average of partial fuzzy evaluations is used. The weights (crisp, not fuzzy) express shares of particular partial evaluations in the aggregated evaluation. Fuzzy evaluations on all levels of the goals tree express fuzzy degrees of fulfillment of the corresponding goals. Publicly available version of NEFRIT does not make it possible to use a fuzzy expert system for evaluation. This software was originally developed for the Czech National Bank (decision making about granting a credit). Further, it was used e.g. by the Czech Tennis Association, the Czech Basketball Association and in other institutions. Nowadays it is tested by the Supreme Audit Office of the Czech Republic. The successor of NEFRIT, in terms of the used theoretical basis, is the FuzzME software.

The FuzzME software (**Fuzz**y models of **M**ultiple-criteria **E**valuation), presented in this paper, is based on the theoretical concept of evaluation which is very close to the original Zadeh's ideas. Similarly to his paper [1], the evaluations of alternatives according to particular criteria represent their degrees of fulfillment of the corresponding partial goals. Besides evaluations expressed by real numbers in $[0,1]$, fuzzy evaluations modeled by fuzzy numbers on the same interval are employed in the software. They represent, analogously, the fuzzy degrees of fulfillment of the partial goals which are connected to the criteria. Resulting fuzzy evaluations, which are obtained by aggregation, have a similar clear interpretation. This theoretical approach to (fuzzy) evaluation was published in the book [8] and in the paper [7] and is used also in NEFRIT.

In contrast with NEFRIT, the aggregation is not limited only to simple weighted average method. The FuzzME software also enables to use the fuzzy OWA operator for the aggregation or to define evaluating function by a fuzzy rule base.

For the aggregation of the partial evaluations by the method of weighted average, fuzzy weights can be used (in contrast to NEFRIT which works only with crisp weights). The theory of normalized fuzzy weights, ways of their setting (including a method for removing potential inconsistence) and algorithm for calculation of the fuzzy weighted average are taken from [9].

Another fuzzy aggregation operator, available in the FuzzME software, is a fuzzified OWA operator. Again, it works with normalized fuzzy weights. The fuzzy OWA operator and the used algorithm for its calculation are described in [10].

In the FuzzME software, multiple-criteria evaluating functions can also be defined by means of fuzzy rule bases. Three algorithms are offered for the approximate reasoning - the standard Mamdani algorithm and two modified Sugeno algorithms (Sugeno-WA and Sugeno-WOWA). The advantage of this software is that all of these types of aggregation can be arbitrarily combined in the same goals tree.

There are also software products for multiple-criteria decision making based on other mathematical methods but they are usually designed for solving a particular assignment. Fuzzy toolboxes of general mathematical systems such as Matlab can be used for multiple-criteria decision making, too. But our investigation by means of Internet did not result software fully comparable to FuzzME. Its universality and comprehensiveness make it unique.

2. Preliminaries

A fuzzy set A on a universal set X is characterized by its membership function $A : X \rightarrow [0, 1]$. $Ker\ A$ denotes a kernel of A, $Ker\ A = \{x \in X \mid A(x) = 1\}$. For any $\alpha \in [0, 1]$, A_α denotes an α-cut of A, $A_\alpha = \{x \in X \mid A(x) \geq \alpha\}$. A support of A is defined as $Supp\ A = \{x \in X \mid A(x) > 0\}$. The symbol $hgt\ A$ denotes a height of the fuzzy set A, $hgt\ A = \sup \{A(x) \mid x \in X\}$. An intersection and a union of the fuzzy sets A and B on X are defined for all $x \in X$ by the following formulas: $(A \cap B)(x) = \min \{A(x), B(x)\}$, $(A \cup B)(x) = \max \{A(x), B(x)\}$.

A fuzzy number is a fuzzy set C on the set of all real numbers $\Re$ which satisfies the following conditions: a) the kernel of C, $Ker\ C$, is not empty, b) the α-cuts of C, C_α, are closed intervals for all $\alpha \in (0, 1]$, c) the support of C, $Supp\ C$, is bounded. A fuzzy number C is called to be defined on $[a, b]$, if $Supp\ C \subseteq [a, b]$.

Real numbers $c^1 \leq c^2 \leq c^3 \leq c^4$ are called significant values of the fuzzy number C if the following holds: $[c^1, c^4] = Cl(Supp\ C), [c^2, c^3] = Ker\ C$, where $Cl(Supp\ C)$ denotes a closure of $Supp\ C$.

Any fuzzy number C can be characterized by a pair of functions $\underline{c} : [0,1] \to \Re$, $\overline{c} : [0,1] \to \Re$ which are defined by the following formulas: $C_\alpha = [\underline{c}(\alpha), \overline{c}(\alpha)]$ for all $\alpha \in (0,1]$, and $Cl(Supp\ C) = [\underline{c}(0), \overline{c}(0)]$. The fuzzy number C is called to be linear if both the functions $\underline{c}$, $\overline{c}$ are linear. A linear fuzzy number is fully determined by its significant values because $\underline{c}(\alpha) = (c_2 - c_1) \cdot \alpha + c_1$, $\overline{c}(\alpha) = (c_3 - c_4) \cdot \alpha + c_4$. For that reason, we can denote it as $C = (c^1, c^2, c^3, c^4)$.

An ordering of fuzzy numbers is defined as follows: a fuzzy number C is greater than or equal to a fuzzy number D, if $C_\alpha \geq D_\alpha$ for all $\alpha \in (0,1]$.

A fuzzy scale makes it possible to represent a closed interval of real numbers by a finite set of fuzzy numbers. Let $T_1, T_2, ..., T_s$ be fuzzy numbers defined on $[a,b]$, forming a fuzzy partition on the interval, i.e., for all $x \in [a,b]$ the following holds

$$\sum_{i=1}^{s} T_i(x) = 1, \tag{1}$$

then the set of the fuzzy numbers can be linearly ordered (see [8]). If the fuzzy numbers $T_1, T_2, ..., T_s$ are defined on $[a,b]$, form a fuzzy partition on the interval and are numbered according to their linear ordering, then they are said to form a fuzzy scale on $[a,b]$.

An uncertain division of the whole into m parts can be modeled by normalized fuzzy weights. Fuzzy numbers $V_1, ..., V_m$ defined on $[0,1]$ are normalized fuzzy weights if for any $i \in \{1, ..., m\}$ and any $\alpha \in (0,1]$ it holds that for any $v_i \in V_{i\alpha}$ there exist $v_j \in V_{j\alpha},\ j = 1, ..., m,\ j \neq i$, such that

$$v_i + \sum_{j=1, j\neq i}^{m} v_j = 1. \tag{2}$$

3. The FuzzME software

The mathematical models of the FuzzME software are based primarily on the theory and methods of multiple-criteria evaluation that were published in [8] and [7]. The theory of normalized fuzzy weights, the definition of fuzzy weighted average, and the algorithm for its computation were taken from [9], [11] and [12]. The fuzzified OWA operator and the algorithm for its calculation published in [10] are also used in the software.

In the FuzzME software, the basic structure of the fuzzy model of multiple-criteria evaluation is expressed by a goals tree. The root of the tree represents the overall goal of evaluation and each branch corresponds to a partial goal. The

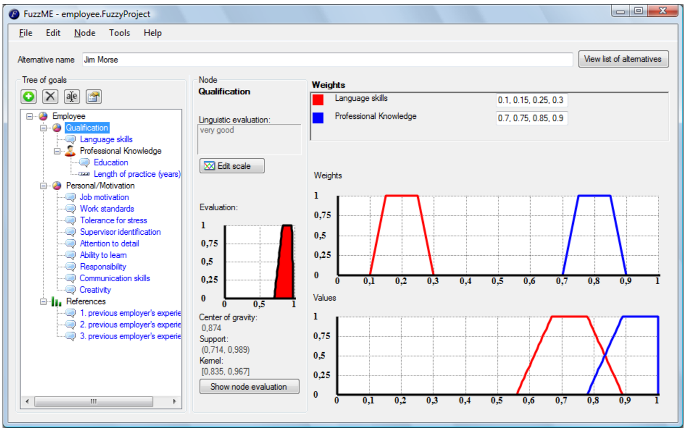

FIGURE 1. The main window of the software

goals at the ends of branches are connected either with quantitative or qualitative criteria.

When an alternative is evaluated, evaluations with respect to criteria connected with the terminal branches are calculated first. Independently of the criterion type, each of the evaluations is described by a fuzzy number defined on the interval $[0, 1]$. It expresses the fuzzy degree of fulfillment of the corresponding partial goal.

These partial fuzzy evaluations are then aggregated according to the defined type of the tree node. Three types of aggregation are available: a fuzzy weighted average (fuzzy WA), an ordered fuzzy weighted average (fuzzy OWA) or aggregation by means of a fuzzy expert system. For aggregation by fuzzy weighted average or ordered fuzzy weighted average, normalized fuzzy weights must be set. The weights express uncertain shares of the partial evaluations in the aggregated one. For the fuzzy expert system, the fuzzy rule base must be defined and an inference algorithm must be chosen (the Mamdani algorithm, the Sugeno-WA or the Sugeno-WOWA algorithm of approximate reasoning).

The overall evaluation reflects the degree of fulfillment of the main goal. A verbal description of the overall evaluation can be obtained by means of the implemented linguistic approximation algorithm.

The overall evaluations can be compared within the frame of a given set of alternatives. By this comparison the best of the alternatives can be chosen. That is why the FuzzME software can be also used as a decision support system.

The import and export of data is supported by the software, too. The FuzzME software is available in the Czech and English versions.

3.1. **Goals tree.** Goals trees represent the basic structure of fuzzy models of multiple-criteria evaluation in the FuzzME software. When a goals tree is designed, the main goal is consecutively divided into goals of progressively lower levels. The process of division is stopped when such goals are reached whose fulfillment can be assessed by means of some known characteristics of alternatives (i.e. quantitative or qualitative criteria).

The design of a tree structure in the goals-tree editor is the first step in forming a fuzzy evaluation model in FuzzME. In the next step, the type of each node in the tree must be specified. For the nodes at the ends of tree branches the user defines if the node is connected with a quantitative or qualitative criterion. For the other nodes he/she sets the type of aggregation - fuzzy weighted average, ordered fuzzy weighted average or fuzzy expert system.

3.2. **Criteria of evaluation.** In the models of evaluation created by the FuzzME software, qualitative and quantitative criteria can be combined arbitrarily.

3.2.1. *Qualitative criteria.* According to qualitative criteria, alternatives are evaluated verbally, by means of values of linguistic variables of special kinds - linguistic scales, extended linguistic scales and linguistic scales with intermediate values.

A linguistic variable is defined as a quintuple $(\mathcal{V}, \mathcal{T}(\mathcal{V}), X, G, M)$, where $\mathcal{V}$ is a name of the variable, $\mathcal{T}(\mathcal{V})$ is a set of its linguistic values, X is a universal set on which the meanings of the linguistic values are defined, G is a syntactic rule for generating values in $\mathcal{T}(\mathcal{V})$, and M is a semantic rule which maps each linguistic value $\mathcal{C} \in \mathcal{T}(\mathcal{V})$ to its mathematical meaning, $C = M(\mathcal{C})$, which is a fuzzy set on X.

A linguistic scale on $[a, b]$ is a special case of the linguistic variable $(\mathcal{V}, \mathcal{T}(\mathcal{V}), X, G, M)$, where $X = [a, b]$, $\mathcal{T}(\mathcal{V}) = \{\mathcal{T}_1, \mathcal{T}_2, ..., \mathcal{T}_s\}$ and the meanings of the linguistic values $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_s$ are modeled by fuzzy numbers $T_1, T_2, \ldots, T_s$ which form a fuzzy scale on $[a, b]$. As the set of linguistic values of the scale is defined explicitly, it is not necessary to include the grammar G into the scale notation.

In the FuzzME software, the user defines a linguistic scale for each qualitative criterion in the fuzzy-scale editor. For example, the linguistic scale *communication skills of an employee* can contain linguistic values *inadequate*, *adequate*, *satisfying*, *good* and *very good.* The evaluating linguistic scale is usually defined on $[0, 1]$; in other cases, it has to be transformed to this interval.

The extended linguistic scale contains, besides elementary terms of the original scale, $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_s$, also derived terms in the form $\mathcal{T}_i$ *to* $\mathcal{T}_j$, where $i < j, i, j \in \{1, 2, \ldots, s\}$. For example, the user can evaluate *communication skills of an employee* by the linguistic term *satisfying to very good.* The meaning of the linguistic value $\mathcal{T}_i$ *to* $\mathcal{T}_j$ is modeled by $T_i \cup_L T_{i+1} \cup_L \cdots \cup_L T_j$, where $\cup_L$ denotes the

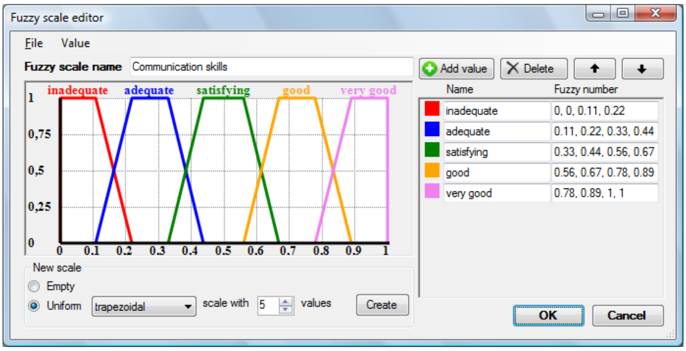

FIGURE 2. Linguistic scale editor

union of fuzzy sets based on the Lukasiewicz disjunction; e.g. $(T_i \cup_L T_{i+1})(x) = \min\{1, T_i(x) + T_{i+1}(x)\}$ for all $x \in \Re$.

The linguistic scale with intermediate values is the original linguistic scale enriched with derived terms *between* $\mathcal{T}_i$ *and* $\mathcal{T}_{i+1}$, $i \in \{1, 2, \ldots, s-1\}$. The meaning of the derived term *between* $\mathcal{T}_i$ *and* $\mathcal{T}_{i+1}$ is modeled by the arithmetic average of the fuzzy numbers T_i and T_{i+1}.

In the FuzzME software, the user evaluates a given alternative according to a qualitative criterion by selecting a proper linguistic evaluation from a drop-down list box. He/she can choose the value from a standard linguistic scale, extended scale or scale with intermediate values.

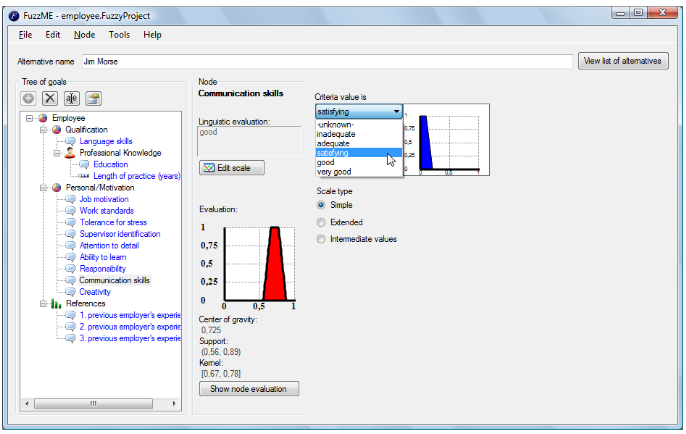

FIGURE 3. Choosing the value of a qualitative criterion

The three mentioned structures of linguistic values are also applied when resulting fuzzy evaluations are approximated linguistically.

3.2.2. *Quantitative criteria.* The evaluation of an alternative with respect to a quantitative criterion is calculated from the measured value of the criterion by means of the evaluating function expertly defined for the criterion. The evaluating function is the membership function of the corresponding partial goal. The FuzzME software admits both crisp and fuzzy values of quantitative criteria. The fuzzy values represent inaccurate measurements or expert estimations of the criteria values. In the case of a fuzzy value, the corresponding partial fuzzy evaluation is calculated by the extension principle.

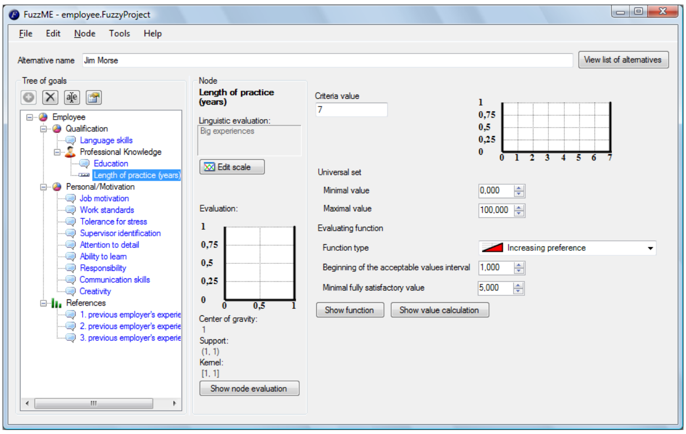

Figure 4. A quantitative criterion

In the FuzzME software, the evaluating function of a quantitative criterion is formally set by means of a fuzzy number. For example, if the evaluating function is defined by a linear fuzzy number $F = (f_1, f_2, f_3, f_4)$, then f_1 is the lower limit of all at least partly acceptable values of the criterion, f_2 is the lower limit of its fully satisfactory values, f_3 is the upper limit of the fully satisfactory values, and f_4 is the upper limit of the acceptable values.

For example, when a company wants to hire a new employee, the candidates are evaluated according to the length of their practice. Evaluating function for this quantitative criterion can be defined by a linear fuzzy number with significant values 2, 5, 100, 100. In that case, less than 2 years of practice are not satisfying at all. For the length of practice from 2 to 5 years the satisfaction of the company is growing linearly. More than 5 years of practice is fully satisfactory from the company??s point of view. Values greater than 100 are not supposed to occur.

This way we can define a monotonous evaluating function, which is the most common in the evaluating models, by a fuzzy number.

In the FuzzME software, this process is simplified for the user. It is necessary to chooses just the type of the evaluating function (increasing preference, decreasing preference or preference of a selected value) and set only some of the significant values.

3.3. **Methods of aggregation of partial evaluations.** The calculated partial fuzzy evaluations are then consecutively aggregated according to the structure of the goals tree. With respect to the defined type of the tree node, the fuzzy weighted average method, the ordered fuzzy weighted average method or the fuzzy expert system method is used for the aggregation. Each of the aggregation methods is suitable for a different situation:

The fuzzy weighted average is used if the goal corresponding with the node of interest is fully decomposed into disjunctive goals of the lower level. The normalized fuzzy weights represent uncertain shares of these lower-level goals in the goal corresponding with the considered node.

Again, the ordered fuzzy weighted average requires that the goal corresponding with the given node is decomposed into disjunctive goals of the lower level. In contrast to the fuzzy weighted average, the usage of this aggregation operator supposes special user's requirements concerning the structure of partial fuzzy evaluations. The normalized fuzzy weights again represent uncertain shares of the partial evaluations in the aggregated one. But the normalized fuzzy weights are not linked to the individual partial goals; the correspondence between the weights and the partial evaluations is given by the ordering of partial evaluations of the alternative of interest. It means, evaluations with respect to the same partial goal can have different weights for different alternatives.

If the relationship between the evaluations of the lower level and the evaluation corresponding with the given node is more complex (if neither of the two previous methods can be used), and if expert knowledge about the relationship is available, then the aggregation function is described by a fuzzy rule base of a fuzzy expert system. The approximate reasoning is used to calculate the resulting evaluation. In particular, evaluating function described by a fuzzy expert system is used if the fulfillment of a goal at the end of a tree branch depends on several mutually dependent criteria (i.e., if combinations of criteria values bring synergic or disynergic effects to the resulting multiple-criteria evaluation).

3.3.1. *Aggregation by the fuzzy weighted average method.* If the fuzzy weighted average is used for aggregation of partial fuzzy evaluations, then the uncertain weights of the corresponding partial goals, which express their shares in the superior goal, must be set. To define consistent uncertain weights, a special structure of fuzzy numbers, normalized fuzzy weights, must be used.

In the FuzzME software, both real and fuzzy normalized weights can be used. Normalized real weights, i.e., real numbers $v_1, ..., v_m$, $v_j \geq 0$, $j = 1, ..., m$, $\sum_{j=1}^{m} v_j = 1$, represent a special case of the normalized fuzzy weights.

The fuzzy weighted average of the partial fuzzy evaluations, i.e., of fuzzy numbers $U_1, ..., U_m$ defined on $[0, 1]$, with the normalized fuzzy weights $V_1, \ldots, V_m$, is a fuzzy number U on $[0, 1]$ whose membership function is defined for any $u \in [0, 1]$ as follows

$$U(u) = \max\{\min\{V_1(v_1), ..., V_m(v_m), U_1(u_1), ..., U_m(u_m)\}$$

$$| \sum_{i=1}^{m} v_i u_i = u, \sum_{i=1}^{m} v_i = 1, \; v_i, u_i \in [0, 1], i = 1, ..., m\}. \tag{3}$$

For an expert who sets the fuzzy weights, it is not so easy to satisfy the condition of normality. That is why the FuzzME software allows to set only an approximation to the normalized fuzzy weights - fuzzy numbers $W_1, ..., W_m$ on $[0, 1]$ satisfying the following weaker condition

$$\exists w_i \in Ker\, W_i, \; i = 1, ..., n : \sum_{i=1}^{n} w_i = 1. \tag{4}$$

The software removes the potential inconsistence in $W_1, ..., W_m$ and derives the normalized fuzzy weights $V_1, ..., V_m$ from them.

The structure of normalized fuzzy weights and the fuzzy weighted average operation are studied in detail in [9], [11] and [12]. Conditions for verifying normality of fuzzy weights, an algorithm for normalization of fuzzy weights satisfying the condition (4), and an algorithm for calculating fuzzy weighted average, which are all used in the FuzzME software, can be found there. Let us notice, that the used algorithm of fuzzy weighted average calculation is very effective.

3.3.2. *Aggregation by the ordered fuzzy weighted average.* The fuzzy OWA operator is used in case that the evaluator has special requirements concerning the structure of the partial evaluation. For example, he/she does not want any partial goal to be satisfied poorly. Then the weight of the minimum partial evaluation of any alternative equals 1, and the weights of all its other partial evaluations equal 0. The aggregated fuzzy evaluations then represent the guaranteed fuzzy degrees of fulfillment of all the partial goals (the fuzzy MINIMAX method). Another example of the fuzzy OWA operator usage could be the evaluation of subjects who can choose in which of the three areas they will be mostly involved. The evaluation algorithm should take into account their right of choice. Then, e.g., the results in the area where the subject performs best contribute to the overall evaluation by about one half, results from the second area by one third and results from the area in which the subject was least involved contribute to the overall evaluation only by one sixth. A practical application of such a fuzzy evaluation model could be the overall evaluation of the academic staff with respect to their results in the areas of research, education, and management of education and science.

The ordered fuzzy weighted average represents a fuzzification of the crisp OWA operator by means of the extension principle. Uncertain weights are modeled by normalized fuzzy weights as in the case of fuzzy weighted average.

The following notation will be used to define the ordered fuzzy weighted average: if $(x_1, ..., x_m)$ is a vector of real numbers, then $(x^{(1)}, ..., x^{(m)})$ is a vector in which for all $j \in \{1, \dots, m\}$, $x^{(j)}$ is the j-th greatest number of $x_1, ..., x_m$.

The ordered fuzzy weighted average of the partial fuzzy evaluations, i.e., of fuzzy numbers $U_1, ..., U_m$ defined on $[0,1]$, with the normalized fuzzy weights $V_1, \dots, V_m$, is a fuzzy number U on $[0,1]$ whose membership function is defined for any $u \in [0,1]$ as follows

$$U(u) = \max\{\min\{V_1(v_1), ..., V_m(v_m), U_1(u_1), ..., U_m(u_m)\} \tag{5}$$
$$\mid \sum_{i=1}^{m} v_i u^{(i)} = u,\ \sum_{i=1}^{m} v_i = 1,\ v_i, u_i \in [0,1], i = 1, ..., m\}.$$

The algorithm used to calculate the ordered fuzzy weighted average in the FuzzME software was taken from [10], where fuzzification of the OWA operator is described in detail. The used algorithm is an analogy to the one used for the fuzzy weighted average.

3.3.3. *Aggregation by the fuzzy expert system.* The fuzzy expert system is used if the relationship between the criteria (or the partial evaluations) and the overall evaluation is complicated. Theoretically, it is possible to model, with an arbitrary precision, any Borel measurable function by means of a fuzzy rule base (properties of Mamdani and Sugeno fuzzy controllers, see e.g. [13]) In reality, the quality of the approximation is limited by the expert's knowledge of the relationship.

If the fuzzy rule base models the relation between values of criteria and the fulfillment of the corresponding partial goal, then the evaluation function is of the following form

$$\begin{aligned} &\text{If } \mathcal{C}_1 \text{ is } \mathcal{A}_{1,1} \text{ and } \dots \text{ and } \mathcal{C}_m \text{ is } \mathcal{A}_{1,m}, \text{then } \mathcal{E} \text{ is } \mathcal{U}_1 \\ &\text{If } \mathcal{C}_1 \text{ is } \mathcal{A}_{2,1} \text{ and } \dots \text{ and } \mathcal{C}_m \text{ is } \mathcal{A}_{2,m}, \text{then } \mathcal{E} \text{ is } \mathcal{U}_2 \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &\text{If } \mathcal{C}_1 \text{ is } \mathcal{A}_{n,1} \text{ and } \dots \text{ and } \mathcal{C}_m \text{ is } \mathcal{A}_{n,m}, \text{then } \mathcal{E} \text{ is } \mathcal{U}_n \end{aligned} \tag{6}$$

where for $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$, $(\mathcal{C}_j, \mathcal{T}(\mathcal{C}_j), V_j, M_j)$ are linguistic scales representing the criteria, $\mathcal{A}_{i,j} \in \mathcal{T}(\mathcal{C}_j)$ are their linguistic values, $(\mathcal{E}, \mathcal{T}(\mathcal{E}), [0,1], M_e)$

is a linguistic scale representing the evaluation of alternatives and $\mathcal{U}_i \in \mathcal{T}(\mathcal{E})$ are its linguistic values.

In the FuzzME software, rule bases are defined expertly. The user defines such a rule base by assigning a linguistic evaluation to each possible combination of linguistic values of criteria.

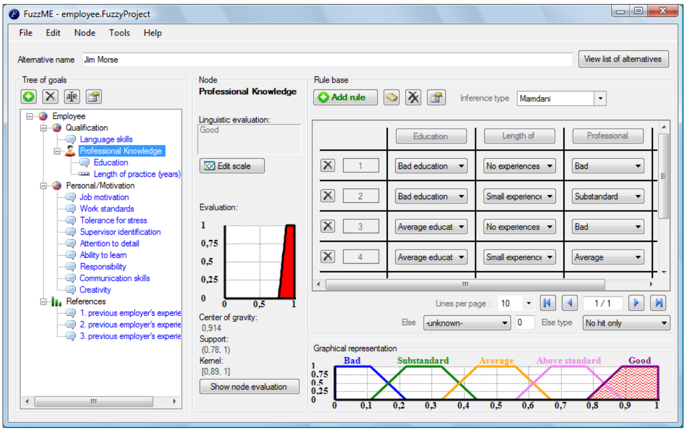

FIGURE 5. Rule base editor

For given values of criteria, a resulting fuzzy evaluation is calculated either by the Mamdani fuzzy inference algorithm, by the Sugeno-WA or the Sugeno-WOWA inference.

In the case of the Mamdani fuzzy inference, the degree h_i of correspondence between the given m-tuple of fuzzy values $(A_1^{'}, A_2^{'}, \ldots, A_m^{'})$ of criteria and the mathematical meaning of the left-hand side of the i-th rule is calculated for any $i = 1, \ldots, n$ in the following way

$$h_i = \min \{hgt(A_1^{'} \cap A_{i,1}), \ldots, hgt(A_m^{'} \cap A_{i,m})\}. \tag{7}$$

Then for each of the rules, the output fuzzy value $U_i^{'}$, $i = 1, \ldots, n$, corresponding to the given input fuzzy values, is calculated as follows

$$\forall y \in [0, 1] : U_i^{'}(y) = \min \{h_i, U_i(y)\}. \tag{8}$$

The final fuzzy evaluation of the alternative is given as the union of all the fuzzy evaluations that were calculated for the particular rules in the previous step, i.e.,

$$U^{'} = \bigcup_{i=1}^{n} U_i^{'}. \tag{9}$$

Generally, the result obtained by the Mamdani inference algorithm need not be a fuzzy number. So, for further calculations within the fuzzy model, it must be approximated by a fuzzy number.

The advantage of the generalized Sugeno inference algorithm (see [8]) is that the result is always a fuzzy number. Two version of this algorithm were implemented - Sugeno-WA and, more advanced, Sugeno-WOWA.

In its first step, the degrees of correspondence h_i, $i = 1, \ldots, n$, are calculated in the same way as in the Mamdani fuzzy inference algorithm.

In Sugeno-WA algorithm, the resulting fuzzy evaluation U is then computed as a weighted average of the fuzzy evaluations $U_i,\ i = 1, 2, \ldots, n$, which model the mathematical meanings of linguistic evaluations on the right-hand sides of the rules, with the weights h_i. This is done by the following formula

$$U = \frac{\sum\limits_{i=1}^{n} h_i.U_i}{\sum\limits_{i=1}^{n} h_i}. \tag{10}$$

The expert chooses values on the right-hand sides of each rule from the linguistic fuzzy scale $(\mathcal{E}, \mathcal{T}(\mathcal{E}), [0,1], M_e)$. We can see that the result can be also obtained as a weighted average of the fuzzy numbers which model the meaning of all values of this scale. Let $E_1, ...E_k$ be those fuzzy numbers, i.e.

$$E_i = M(\mathcal{E}_i), \text{ where } \mathcal{E}_i \in \mathcal{T}(\mathcal{E}),\ i \in \{1, .., k\}. \tag{11}$$

We can assume that those fuzzy numbers are numbered according their ordering from the greatest to the lowest one, i.e., $E_i > E_{i+1}$ for $i \in \{1, ..., k-1\}$

Let $A_1, ..., A_k$ be sets of indices such that A_i contains indices of all rules which have E_i on their right-hand side, i.e.

$$A_i = \{j \in \{1, .., n\} \mid U_j = E_i\}, i = 1, ..., k \text{ where } U_j = M(\mathcal{U}_j). \tag{12}$$

The weights $w'_1, ..., w'_k \in \Re$, which correspond to the values of the linguistic scale $\mathcal{E}$, are calculated, for every i=1,..k, as follows

$$w'_i = \sum_{j \in A_i} h_j \tag{13}$$

and for the further calculations they are normalized:

$$w_i = \frac{w'_i}{\sum_{j=1}^{k} w'_j}. \tag{14}$$

The resulting evaluation of Sugeno-WA inference algorithm can be then expressed as

$$U = \sum_{i=1}^{k} w_i E_i. \tag{15}$$

Sugeno-WOWA algorithm works in the similar way but, instead of weighted average, weighted OWA operator was used. Weighted OWA operator is described in [14]. This operator uses two sets of weights. Weights w_i are the same as in the case of Sugeno-WA. The second set of weights, p_i, is defined by the expert. This gives him/her possibility to specify how important is each value of the scale for the resulting evaluation. Implementation of this inference system was motivated by the real application of this software. A risk rate was calculated by a fuzzy expert system. Expert set significantly greater weight to linguistic value "high risk" than to the value "medium risk". This causes that a single rule that estimated the risk to be high is taken much more seriously than a rule which estimated it to be just medium. So the evaluation algorithm behaves according to the expert's needs because it respects his/her preferences defined by the weights p_i.

The resulting evaluation of Sugeno-WOWA inference algorithm is calculated as follows

$$U = \sum_{i=1}^{k} \omega_i E_i, \tag{16}$$

where the weight ω_i is defined as

$$\omega_i = f(\sum_{j \leq i} w_j) - f(\sum_{j < i} w_j), \tag{17}$$

the weights w_i are the same as in Sugeno-WA algorithm and f is a nondecreasing piecewise linear function that is determined by the following points

$$\{(0,0)\} \ \cup \ \{(i/k, \sum_{j \leq i} p_j)\}_{i=1,\ldots,k}. \tag{18}$$

In case that the weights p_i are uniform (all scale values have the same weight), the result will be the same as the result calculated by Sugeno-WA. The fact that the values of the scale are ordered simplifies the previous formula. Definition of weighted OWA for more general cases can be found in [14].

3.4. **Overall fuzzy evaluations, the optimum alternative.** The final result of the consecutive aggregation of the partial fuzzy evaluations is an overall fuzzy evaluation of the given alternative. The obtained overall fuzzy evaluations are fuzzy numbers on $[0,1]$. They express uncertain degrees of fulfillment of the main goal by the particular alternatives.

The FuzzME software compares alternatives according to the centers of gravity of their overall fuzzy evaluations. A center of gravity of a fuzzy number U on $[0,1]$ that is not a real number, is defined as follows

$$t_U = \frac{\int_0^1 U(x).x \, dx}{\int_0^1 U(x) \, dx}. \tag{19}$$

If $U = u$ and $u \in \Re$, then $t_U = u$. In the FuzzME software, the optimum alternative is the one whose overall fuzzy evaluation has the largest center of gravity.

At present, the FuzzME software is aimed above all at solving multiple-criteria evaluation problems. To ensure high performance in choosing the optimum alternative, it will be necessary to include in the software other methods of ordering of the fuzzy evaluations in the future. Some approaches are proposed in [8] and further research in this area is planned.

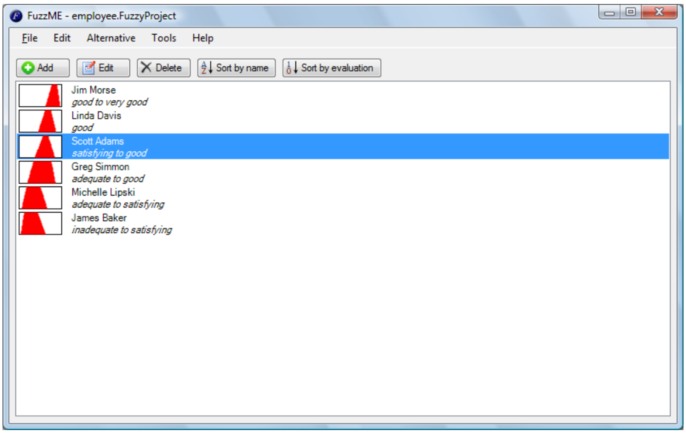

FIGURE 6. A list of alternatives ordered by centers of gravity method

3.5. **Import and export of data.** For fuzzy models of evaluation created in the frame of the program FuzzME, the criteria values of alternatives can be either set directly or imported e.g. from Excel. Similarly resulting evaluations can be exported to the Excel for their further processing.

4. EXAMPLE

The possibilities of this software can be demonstrated on a simple example. Let us consider a company which is going to hire a new employee. There are several candidates and the company naturally wants to select the best of them.

In this example, there are six candidates which are evaluated according to fifteen criteria. Both qualitative and quantitative criteria were used.

For the most of the tree nodes, the fuzzy weighted average was sufficient for the aggregation. One of the exceptions was aggregation of the candidate's references. In this example, it is assumed that the company will try to ask last three of the candidate's previous employers on their experiences with this candidate. The company is careful and wants the worst of these three evaluations to have the greatest weight. But the other two evaluations should be also taken into account. This can be easily solved by fuzzy OWA operator.

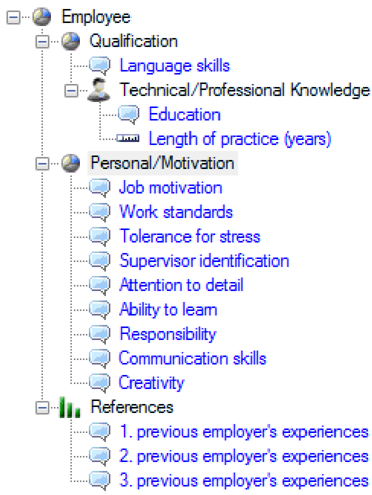

FIGURE 7. The goals tree used in this example

For the evaluation of candidate's technical/professional knowledge a fuzzy expert system was used. This evaluation is obtained from evaluation of candidate's education level and his/her length of practice. Naturally, if the candidate has lots of years of practice then the education level is irrelevant. On the other hand, if the candidate has only small or no practice, the education level should be taken into account. This relationship is too complicated for fuzzy weighted average or fuzzy OWA, but can be easily modeled by a fuzzy rule base.

This simple example shows the advantage over other software products for fuzzy evaluation and decision making. The user has freedom in choosing the aggregation method and they can be arbitrarily combined in the same goals tree.

The FuzzME demo version with this example can be downloaded at `http://FuzzME.wz.cz/`.

5. Conclusion

The FuzzME software makes it possible to create and use fuzzy models of multiple criteria evaluation in the user-friendly way. It has several positive features. The essential one is the solid theoretical basis of the methods contained in the program. The mathematical potential of the software is a result of many years of research. The implemented methods were tested on real problems.

In the FuzzME software, several new methods, algorithms and tools of fuzzy modeling were implemented, e.g.: a structure of normalized fuzzy weights, fuzzy weighted average and ordered fuzzy weighted average operations and algorithms for their calculation and Sugeno-WOWA inference algorithm.

Well-elaborated theoretical basis of the FuzzME software provides a clear interpretation of all steps of the evaluation process and brings understanding of methodology to the user.

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