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Intuitionistic fuzzy sets – two and three term representations in the context of a Hausdorff distance

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Dedicated to the 75th birthday of Beloslav Riečan

Abstract

We consider here the two term and three term representations of Atanassov's intuitionistic fuzzy sets (A-IFSs, for short) in the context of the Hausdorff distance based on the Hamming metric. Especially, we pay attention to the consistency of the metric used and the essence of the Hausdorff distances. We also consider the same problem for the interval-valued fuzzy sets. It is shown that the essence of solutions obtained is different for the case of the A-IFSs and interval-valued fuzzy sets. In other words, the two term representation of A-IFSs (which makes the A-IFSs to boil down to the interval-valued fuzzy sets) does not work here (on the contrary to three term representation).

Keywords Intuitionistic fuzzy sets, Hausdorff distance

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1 Introduction

One of the most important measures are distances which are widely used both in theoretical considerations and for practical purposes in many areas. It is not possible to overestimate their importance also in the context of fuzzy sets (Zadeh [45]) or their generalizations, eg., the A-IFSs. Distances are necessary in analyses related to the entropy, similarity, when making group decisions, calculating degrees of soft consensus, in classification, pattern recognition, etc.

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Distances between the A-IFSs are calculated in the literature in two ways, using two terms only, i.e. the degree of membership and non-membership (e.g., Atanassov [4]) or all three terms. i.e. the membership and non-membership degrees and the hesitation margin (e.g., Szmidt and Kacprzyk [28], [35], Tasseva et al. [43], Atanassov et al. [5], Szmidt and Baldwin [22], [23], Deng-Feng [8], Tan and Zhang [42], Narukawa and Torra [13])). Mathematically, both ways are correct from the point of view of just the formal conditions concerning distances (all properties are fulfilled in both cases). However, when semantics come to play, one cannot say that both ways are equal when assessing the results obtained by the two approaches. In this paper we will consider one of such situations related to the calculating a Hausdorff distance using the two approaches to represent the A-IFSs.

The Hausdorff distances (cf. Grünbaum [9]) play an important role in practical applications, notably in image matching, image analysis, motion tracking, visual navigation of robots, computer-assisted surgery and so on (cf. e.g., Huttenlocher et al. [10], Huttenlocher and Rucklidge [11], Olson [14], Peitgen et al. [15], Rucklidge [17]-[21]). The definition of the Hausdorff distances is simple but the calculations needed to solve the real problems are complex. As a result the efficiency of the algorithms for computing the Hausdorff distances may be crucial and the use of some approximations may be relevant and useful (e.g, Aichholzer [1], Atallah [2], Huttenlocher et al. [10], Preparata and Shamos [16], Rucklidge [21], Veltkamp [44]).

The formulas proposed for calculating the distances should first of all be formally correct. It is the motivation of this paper. Namely, we consider the results of using the Hamming distances between the A-IFSs calculated in two possible ways - taking into account the two term representation (the membership and non-membership values) of A-IFSs, and next - taking into account the three term representation (the membership, non-membership values, and hesitation margin) of A-IFSs. We will verify if the resulting distances fulfill the properties of the Hausdorff distances.

We also consider the problem of calculating the Hausdorff distance based on the Hamming metric for the interval-valued fuzzy sets. We prove that the formulas that are effective and efficient for interval-valued fuzzy sets do not work well in the case of A-IFSs.

2 Brief introduction to the A-IFSs

One of the generalizations of a fuzzy set in X (Zadeh [45]) , given by

$$A^{'} = \{< x, \mu_{A'}(x) > | x \in X\} \tag{2.1}$$

where $\mu_{A'}(x) \in [0,1]$ is the membership function of the fuzzy set $A^{'}$, is the intuitionistic fuzzy set, or A-IFS, for short (Atanassov [3], [4]) A given by

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \tag{2.2}$$

where: $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{2.3}$$

and $\mu_A(x)$, $\nu_A(x) \in [0,1]$ denote a degree of membership and a degree of non-membership of $x \in A$, respectively. These degrees may be specified in different ways, and a constructive approach is given by Szmidt and Baldwin [24].

Obviously, each fuzzy set may be represented by the following A-IFS

$$A = \{< x, \mu_{A'}(x), 1 - \mu_{A'}(x) > | x \in X\} \tag{2.4}$$

An additional concept for each A-IFS in X, that is not only an obvious result of (2.2) and (2.3) but which is also relevant for applications, is

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{2.5}$$

a *hesitation margin (an intuitionistic fuzzy index)* of $x \in A$ which expresses a lack of knowledge of whether x belongs to A or not (cf. Atanassov [4]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$.

The hesitation margin turns out to be important while considering the distances (Szmidt and Kacprzyk [26], [28], [35], entropy (Szmidt and Kacprzyk [31], [38]), similarities (Szmidt and Kacprzyk [39]), etc. i.e., measures that play a crucial role in virtually all information processing tasks.

Also, from the point of view of the applications, the hesitation margin is crucial in many areas exemplified by image processing (cf. Bustince et al. [6], [7]), classification of imbalanced and overlapping classes (cf. Szmidt and Kukier [37], [40], [41]), group decision making, negotiations, voting and other situations (cf. Szmidt and Kacprzyk [25], [27], [29], [30], [32], [33], [34], [36]).

In other words, the three term representation of the A-IFSs (taking into account the membership values, non-membership values, and hesitation margins) has already proved to play important role both from the theoretical point of view and applications.

2.1 Distances Between the A-IFSs

In Szmidt and Kacprzyk [28], [35], Szmidt and Baldwin [22], [23], it is shown why in the calculation of distances between the A-IFSs one should use all three terms describing them. Examples of the distances between any two A-IFSs A and B in $X = \{x_1, x_2, \ldots, x_n\}$ while using the three term representation (Szmidt and Kacprzyk [28], Szmidt and Baldwin [22], [23]) may be as follows:

- the normalized Hamming distance:

$$\begin{aligned} l_{IFS}(A,B) &= \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + \\ &+ |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|) \end{aligned} \tag{2.6}$$

- the normalized Euclidean distance:

$$\begin{aligned} e_{IFS}(A,B) = (\frac{1}{2n}\sum_{i=1}^{n}(\mu_A(x_i) - \mu_B(x_i))^2 + \\ + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2)^{\frac{1}{2}} \end{aligned} \tag{2.7}$$

  The values of both distances are from the interval $[0,1]$.

The counterparts of the above distances while using the two term representation of A-IFSs are:

- the normalized Hamming distance:

$$l^{'}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|) \tag{2.8}$$

- the normalized Euclidean distance:

$$q^{'}(A,B) = (\frac{1}{2n}\sum_{i=1}^{n}(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2)^{\frac{1}{2}} \tag{2.9}$$

3 The Hausdorff distance

The Hausdorff distance is *the maximum distance of a set to the nearest point in the other set*. More formal description is given by the following

Definition 3.1. Given two finite sets $A = \{a_1, ..., a_p\}$ and $B = \{b_1, ..., b_q\}$, the Hausdorff distance $H(A, B)$ is defined as:

$$H(A, B) = \max\{h(A, B), h(B, A)\} \tag{3.1}$$

where

$$h(A, B) = \max_{a \in A} \min_{b \in B} d(a, b) \tag{3.2}$$

where:
– a and b are elements of sets A and B respectively,
– $d(a, b)$ is any metric between these elements,
– the two distances $h(A, B)$ and $h(B, A)$ (3.2) are called the directed Hausdorff distances.

The function $h(A, B)$ (the directed Hausdorff distance from A to B) ranks each element of A based on its distance to the nearest element of B, and then the highest ranked element specifies the value of the distance. In general $h(A, B)$ and $h(B, A)$ can be different values (the directed distances are not symmetric).

From Definition 3.1 it follows, that if A and B contain one element each (a_1 and b_1, respectively), the Hausdorff distance is just equal to $d(a_1, b_1)$. In other words, if a formula which is expected to express the Hausdorff distance gives for separate elements the results not consistent with the used metric d (e.g., the Hamming distance, the Euclidean distance, etc.), the formula considered is not a proper definition of the Hausdorff distance.

3.1 The Hausdorff distance between the interval-valued fuzzy sets

The Hausdorff distance between two intervals: $U = [u_1, u_2]$ and $W = [w_1, w_2]$ is (Moore [12]):

$$h(U, W) = \max\{|u_1 - w_1|,\ |u_2 - w_2|\} \tag{3.3}$$

Assuming the two-term representation for the A-IFSs: $A = \{x, \mu_A(x), \nu_A(x)\}$ and $B = \{x, \mu_B(x), \nu_B(x)\}$, we may consider the two A-IFSs, A and B, as two intervals, namely:

$$[\mu_A(x), 1 - \nu_A(x)] \quad \text{and} \quad [\mu_B(x), 1 - \nu_B(x)] \tag{3.4}$$

then

$$h(A, B) = \max\{|\mu_A(x) - \mu_B(x)|,\ |\nu_A(x) - \nu_B(x)|\} \tag{3.5}$$

We will verify later if (3.5) is a properly calculated Hausdorff distance between the A-IFSs while using the Hamming metric.

3.2 Two term representation of A-IFSs and the Hausdorff distance (Hamming metric)

Due to the algorithm of calculating the directed Hausdorff distances, when applying the two term type distance (2.8) for the A-IFSs, we obtain:

$$d_h(A, B) \quad = \quad \frac{1}{n} \sum_{i=1}^{n} max\{|\mu_A(x_i) - \mu_B(x_i)|\,, |\nu_A(x_i) - \nu_B(x_i)|\} \tag{3.6}$$

If the above distance is a properly calculated Hausdorff distance, in the case of degenerated, i.e., one-element sets $A = \{< x, \mu_A(x), \nu_A(x) >\}$ and $B = \{< x, \mu_B(x), \nu_B(x) >\}$, it should give the same results as the the two term type Hamming distance. It means that in the case of the two term type Hamming distance, for degenerated, one element A-IFSs, the following equations should give just the same results:

$$l'(A, B) = \frac{1}{2}(|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)|) \tag{3.7}$$

$$d_h(A, B) = \max\{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|\} \tag{3.8}$$

where (3.7) is the normalized two term type Hamming distance, and (3.8) should be its counterpart Hausdorff distance.

We will verify on a simple example if (3.7) and (3.8) give the same results as they should do following the essence of of the Hausdorff measures.

Example 1
Let consider the following one-element A-IFSs: A, B, $\in X = \{x\}$

$$A \;=\; \{< x, 1, 0 >\}, \quad B = \{< x, \frac{1}{4}, \frac{1}{4} >\} \tag{3.9}$$

The result obtained from (3.8) is:

$$d_h(A, B) \;=\; \max\{|1 - 1/4|, |0 - 1/4|\} = 0.75$$

The counterpart Hamming distance calculated from (3.7) is:

$$l'(A, B) \;=\; 0.5(|1 - 1/4| + |0 - 1/4||) = 0.5$$

i.e. the value of the Hamming distances (3.7) used to propose the Hausdorff measure (3.8), and the value of the resulting Hausdorff distance (3.8) calculated for the separate elements are not consistent (as they should be).

Now we will show that the inconsistencies as shown above occur for an infinite number of other cases.

Let us verify the conditions under which the equation (3.7) and (3.8) give the consistent results, i.e., when for the separate elements we have

$$\begin{aligned} & \frac{1}{2}(|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)|) = \\ = \;& \max\{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|\} \end{aligned} \tag{3.10}$$

Having in mind that

$$\mu_A(x) + \nu_A(x) + \pi_A(x) = 1 \tag{3.11}$$

$$\mu_B(x) + \nu_B(x) + \pi_B(x) = 1 \tag{3.12}$$

from (3.11) and (3.12) we obtain

$$(\mu_A(x) - \mu_B(x)) + (\nu_A(x) - \nu_B(x)) + (\pi_A(x) - \pi_B(x)) = 0 \tag{3.13}$$

It is easy to verify that (3.13) is not fulfilled for all elements belonging to an A-IFSs but for some elements only. The following conditions guarantee that (3.10) is fulfilled

- for $\pi_A(x) - \pi_B(x) = 0$, from (3.13) we have

$$|\mu_A(x) - \mu_B(x)| = |\nu_A(x) - \nu_B(x)| \tag{3.14}$$

and taking into account (3.14), we can express (3.10) in the following way:

$$\begin{aligned} & 0.5(|\mu_A(x) - \mu_B(x)| + |\mu_A(x) - \mu_B(x)|) = \\ = \quad & \max\{|\mu_A(x) - \mu_B(x)|, |\mu_A(x) - \mu_B(x)|\} \end{aligned} \tag{3.15}$$

- if $\pi_A(x) - \pi_B(x) \neq 0$ but the same time

$$\mu_A(x) - \mu_B(x) = \nu_A(x) - \nu_B(x) = -\frac{1}{2}(\pi_A(x) - \pi_B(x)) \tag{3.16}$$

guarantee that (3.10) boils down again to (3.15).

In other words, (3.10) is fulfilled (which means that the Hausdorff measure given by (3.8) is a natural counterpart of (3.7)) only for such elements belonging to an A-IFS, for which some additional conditions are given, like $\pi_A(x) - \pi_B(x) = 0$ or (3.16). But in general, for an infinite numbers of elements, (3.10) is not valid.

In the above context it seems to be a bad idea to try constructing the Hausdorff distance using the two term type Hamming distance between the A-IFSs.

An immediate conclusion is that, relating to the results from Section 3.1, the Hausdorff distance for the A-IFSs can not be constructed in the same way as for the interval-valued fuzzy sets.

3.3 A straightforward generalizations of the Hamming distance based on the Hausdorff metric

Now we will show that applying the three term type Hamming distance for the A-IFSs, we obtain correct (in the sense of Definition 3.1) Hausdorff distance.

Namely, if we calculate the three term type Hamming distance between two degenerated, i.e. one-element A-IFSs, A and B in the spirit of Szmidt and Kacprzyk [28], [35], Szmidt and Baldwin [22], [23], i.e., in the following way:

$$\begin{aligned} l_{IFS}(A,B) \quad = \quad & \frac{1}{2}(|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| + \\ + \quad & |\pi_A(x) - \pi_B(x)|) \end{aligned} \tag{3.17}$$

we can give a counterpart of the above distance in terms of the max function:

$$\begin{aligned} H_3(A,B) \quad = \quad & \max\{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|, \\ , \quad & |\pi_A(x) - \pi_B(x)|\} \end{aligned} \tag{3.18}$$

If $H_3(A,B)$ (3.18) is a properly specified Hausdorff distance (in the sense that for two degenerated, one element A-IFS the result is equal to the metric used), the following condition should be fulfilled:

$$\begin{aligned} & \frac{1}{2}(|\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)|) + |\pi_A(x) - \pi_B(x)|) = \\ = \quad & \max\{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|, |\pi_A(x) - \pi_B(x)|\} \end{aligned} \tag{3.19}$$

Let us verify if (3.19) is valid. Without loss of generality we can assume

$$\begin{aligned} \max \quad & \{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|, |\pi_A(x) - \pi_B(x)|\} = \\ = \quad & |\mu_A(x) - \mu_B(x)| \end{aligned} \tag{3.20}$$

For $|\mu_A(x) - \mu_B(x)|$ fulfilling (3.20), and because of (3.11) and (3.12), we conclude that both $\nu_A(x) - \nu_B(x)$, and $\pi_A(x) - \pi_B(x)$ are of the same sign (both values are either positive or negative). Therefore

$$|\mu_A(x) - \mu_B(x)| = |\nu_A(x) - \nu_B(x)| + |\pi_A(x) - \pi_B(x)| \tag{3.21}$$

Applying (3.21) we can verify that (3.19) always is valid as

$$\begin{aligned} & 0.5\{|\mu_A(x) - \mu_B(x)| + |\mu_A(x) - \mu_B(x)|\} = \\ = \; & \max\{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|, |\pi_A(x) - \pi_B(x)|\} = \\ = \; & |\mu_A(x) - \mu_B(x)| \end{aligned} \tag{3.22}$$

Now we will use the above formulas (3.17) and (3.18) for the data used in Example 1. But now, as we also take into account the hesitation margins $\pi(x)$ (2.5), instead of (3.9) we use the three term, "full" description of the data $\{< x, \mu(x), \nu(x), \pi(x) >\}$, i.e. employing all three functions (the membership, non-membership and hesitation margin) describing the considered A-IFSs:

$$A = \{< x, 1, 0, 0 >\}, \quad B = \{< x, \frac{1}{4}, \frac{1}{4}, \frac{1}{2} >\} \tag{3.23}$$

and obtain from (3.18):

$$H_3(A, B) = \max(|1 - 1/4|, |0 - 1/4|, |0 - 1/2|) = 0.75$$

Now we calculate the counterpart Hamming distances using (3.17) (with all three functions). The results are

$$l_{IFS}(A, B) = 0.5(|1 - 1/4| + |0 - 1/4| + |0 - 1/2|) = 0.75$$

As we can see, the Hausdorff distance (3.18) proposed in this paper (using the memberships, non-memberships and hesitation margins) and the Hamming distance (3.17) give for one-element IFS sets fully consistent results. The same situation occurs in a general case too.

In other words, for the normalized Hamming distance expressed in the spirit of (Szmidt and Kacprzyk [28], [35]) given by (2.6) we can give the following equivalent representation in terms of the max function:

$$\begin{aligned} H_3(A, B) = & \frac{1}{n} \sum_{i=1}^{n} \max \{|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|, \\ & |\pi_A(x_i) - \pi_B(x_i)|\} \end{aligned} \tag{3.24}$$

Unfortunately, it can be easily verified that it is impossible to give the counterpart pairs of the formulas as (2.6) and (3.24) for $r > 1$ in the Minkowski r-metrics ($r = 1$ is the Hamming distance, $r = 2$ is the Euclidean distance, etc.)

For details on other distances between the A-IFSs we refer the interested reader to Szmidt and Kacprzyk [28] and especially [35]. More details are given in [5] and [43].

4 Conclusions

A method for the calculation of Hausdorff distances (based on the Hamming metric) between the A-IFSs is presented and analyzed. The method employs all three terms describing the A-IFSs. The proposed method is both mathematically valid and intuitively appealing (cf. [35]).

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