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Enumeration of coherent configurations of order at most 15

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Abstract
This text describes the computerized enumeration of all coherent configurations of order up to 15, and provides some viewpoints of the results of this enumeration. The main discovery resulting from this enumeration is the unique non-Schurian coherent configuration of order 14. We also provide classification of the association schemes of order at most 30 up to algebraic isomorphism, using the classification up to combinatorial isomorphism of those schemes by Hanaki and Miyamoto.

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1 Introduction

A coherent algebra of order n is a subalgebra of $M_{n\times n}(\mathbb{C})$ that is closed under transposition and Schur-Hadamard products, and contains I_n and J_n (the all one matrix).

A subalgebra $\mathcal{A}$ of $M_{n\times n}(\mathbb{C})$ is a coherent algebra if and only if it has a basis B of $(0,1)$-matrices such that for any $A \in B$, $A^T \in B$, the sum of all matrices in B is J_n, and I_n is in the algebra (or equivalently, I_n is a sum of some matrices in B). B is called the first standard basis of $\mathcal{A}$.

If $B = \{A_1, \ldots, A_r\}$, then the rank of $\mathcal{A}$ is r. $C = \sum iA_i$ is the color matrix of $\mathcal{A}$. More generally, we allow any distinct coefficients.

In relational (or combinatorial) language, $\mathfrak{m} = (\Omega, \mathcal{R})$ is a coherent configuration if $\mathcal{R} = \{R_1, \ldots, R_r\}$, where the R_i are relations over Ω, and $B = \{A(R_i) | 1 \leq i \leq r\}$ is a first standard basis of a coherent algebra. The R_i are called basic relations of $\mathfrak{m}$. R_i is the set of arcs of a directed graph Γ_i. The Γ_i are called basic graphs of $\mathfrak{m}$.

For a basic graph Γ_i, one of the following holds:

1) All arcs are loops; or

2) the graph is simple; or

3) the graph has no undirected edges.

In other words, the corresponding relations are reflexive, symmetric or anti-symmetric.

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Order	1	2	3	4	5	6	7	8
CCs	1	2	4	10	15	38	57	143
Schurian	1	2	4	10	15	38	57	143

Order	9	10	11	12	13	14	15
CCs	228	492	769	1845	2806	6167	9841
Schurian	228	492	769	1845	2806	6166	9839

Table 1. Numbers of coherent configurations of each order

$\Delta = \{(x,x)|x \in \Omega\}$ is a union of some relations of $\mathcal{R}$. This union defines a partition of $\Omega = F_1 \cup \dots \cup F_k$. F_i is called a fiber of $\mathfrak{m}$. For every relation $R_i \in \mathcal{R}$, there exist two fibers F_a, F_b (not necessarily distinct) such that $R_i \subseteq F_a \times F_b$.

The graphs that are within a single fiber are regular. The graphs between two distinct fibers are biregular.

A coherent configuration with a single fiber is called an association scheme.

A color automorphism of a coherent configuration is a permutation $\sigma \in Sym(\Omega)$, such that there exists a permutation $' \in Sym([1,r])$ for which $R_i^\sigma = R_{i'}$ for all $i \in [1,r]$. The set of all color automorphisms of $\mathfrak{m}$ is denoted by $CAut(\mathfrak{m})$.

A (strong) automorphism is a color automorphism for which the $' = id$. The set of all automorphisms of $\mathfrak{m}$ is denoted by $Aut(\mathfrak{m})$.

For more information and details about coherent configurations and association schemes, see e.g. [1, 2, 7].

2 Results

2.1 Enumeration of coherent configurations

The numbers of coherent configurations and of Schurian coherent configurations of each order up to 15 are listed in Table 1.

An interesting consequence of this enumeration can be seen in column 14. There exists a unique non-Schurian coherent configuration of order 14. The existence of such a coherent configuration was an open question until the results of our enumeration, which were announced in [8]. This non-Schurian coherent configuration has two fibers of sizes 6, 8, rank 11 and its automorphism group has rank 12. For a detailed description of this non-Schurian CC, see [8].

The two non-Schurian coherent configurations of order 15 are the well known doubly regular tournament and the order 14 non-Schurian coherent configuration enlarged by a fiber of size 1.

A file with a list of color graphs of the coherent configurations of order up to 15 is available at `http://my.svgalib.org/math-data/ccs1_15n`. The matrices are in GAP ([4]) format. The list does not include coherent configurations with fibers of size 1.

2.2 Algebraic isomorphisms and automorphisms

Recall that a combinatorial (or strong) isomorphism (or simply an isomorphism) of two coherent configurations is a bijection of the underlying sets that maps relations to relations.

An algebraic isomorphism between two coherent configurations is a bijection of the relations of one configuration to the other that preserves the algebraic structure.

An isomorphism of coherent configurations induces naturally an algebraic isomorphism. Algebraic isomorphism that do not arise from combinatorial ones are thus of

Order	#	Classes
16	15	{5,6}, {14,15,16,17}, {18,19}, {20,21}, {32,33}, {49,50}, {54,55}, {58,59}, {77,78,79}, {83,84,85}, {89,90}, {94,95}, {155,156}, {164,165}, {167,168}
19	1	{2,3}
23	1	{2,...,20}
24	20	{53,54,55}, {56,57,58}, {89,...,93}, {94,95,96}, {99,...,103}, {105,106,107,108}, {113,114}, {130,131,132}, {133,134,135}, {163,164,165,166}, {167,...,171}, {175,176,177,178}, {182,...,188}, {189,190,191}, {195,...,201}, {296,297}, {306,307,308}, {382,...,386}, {395,396,397}, {465,466}
25	5	{4,...,11}, {14,15}, {17,18}, {20,21}, {22,23}
26	1	{3,...,12}
27	8	{5,...,378}, {382,383}, {427,428}, {429,430}, {431, 432}, {472,473}, {474,475}, {476,477}
28	5	{5,6,7,8}, {16,...,71}, {74,75}, {109,110}, {175,176}
29	1	{2,...,22}
30	4	{25,26}, {27,28,29,30}, {106,107}, {122,123}

Table 2. Classes of non-isomorphic, algebraically isomorphic association schemes

some interest. In the case of automorphisms, an algebraic automorphism not arising from a (color) automorphism is called a proper algebraic automorphism.

The smallest case of two algebraically isomorphic coherent configurations that are not isomorphic is the pair of two non-isomorphic strongly regular graphs with parameters $(16, 6, 2, 2)$.

The smallest case of a coherent configuration with a proper algebraic automorphism that maps a basic relation to a non-isomorphic basic relation is the doubly regular tournament on 15 points.

As a result of this project, it is now known that the smallest case of a coherent configuration with a proper algebraic automorphism is of order 14 and rank 12. In this case the proper algebraic automorphism exchanges two isomorphic basic relations. A merging of this coherent configuration is the unique non-Schurian coherent configuration of order 14. See [8] for a detailed discussion of this rank 12 coherent configuration.

For comparison, considering only association schemes, and using the list of [9], we found that up to 30 points there are 445 association schemes with proper algebraic automorphisms, see Table 3. Up to 30 points, there are 61 classes of algebraically isomorphic association schemes which are not combinatorially isomorphic, see Table 2.

2.3 Correctness of the results

Since no formal proof that the programs actually implement the correct algorithm is offered, the correctness of the results is not assured. But comparing the results to similar efforts by our predecessors and colleagues (see [8]) may increase the confidence that the results are correct.

For orders up to 8, a different approach may be used (enumeration of subalgebras). The results are the same as the results presented here.

Order	#	Positions
15	1	5
16	5	16, 78, 84, 160, 172
18	1	60
23	18	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19
24	36	53, 54, 55, 58, 72, 130, 131, 132, 134, 163, 164, 165, 166, 168, 169, 170, 175, 176, 177, 178, 188, 261, 272, 273, 276, 277, 383, 384, 385, 386, 458, 593, 597, 598, 600, 601
25	9	4, 5, 6, 7, 8, 9, 10, 27, 34
27	350	5,...,51, 53,...,66, 68,...,73,75,76,77, 79,...,159, 161,...,188, 190,...,207, 209,...,216, 218,...,227, 229,...,266, 268,...,300, 302,...,329, 331,...,341, 343,...,349, 351,352,353, 356,357,358, 360,361, 363, 365,366,367,368, 371,372, 377, 407, 455
28	2	109, 110
29	20	2,...,21
30	3	13, 25, 74

Table 3. List of association schemes with proper algebraic automorphisms

For orders up to 13, the results agree with unpublished results of a similar project by Sven Reichard.

For orders up to 14, no Schurian coherent configuration was missed. A list of Schurian coherent configurations can be easily calculated by GAP for those orders.

3 Algorithm

3.1 The main algorithm

Inducing a coherent configuration on a subset of fibers results again in a coherent configuration. In particular, inducing on a single fiber results in an association scheme. Thus, the color matrix of a coherent configuration with two fibers is of the form

$$C = \left(\begin{array}{c|c} \mathrm{AS}_1 & CB \\ \hline CB^T & \mathrm{AS}_2 \end{array}\right). \qquad \underbrace{\qquad}_{n}\ \underbrace{\qquad}_{m}$$

Here CB is a (color matrix of a) color partition of the complete (directed) bipartite graph $K_{n\to m}$ into biregular subgraphs.

The association schemes of orders up to 30 were enumerated by Hanaki and Miyamoto [9], thus to enumerate all coherent configurations of order n with two fibers, we use Algorithm 1.

For the crucial step of this algorithm, finding all CBs, we use the two step "good sets" method, originally used in `COCO` [3].

In the first step we enumerate all biregular subgraphs H of the complete (directed) graph $K_{i\to j}$ that may be colors of such a CB. There are two requirements:

i) when H is multiplied by a color of AS_1 or AS_2, the result does not split H;

ii) when H is multiplied by H^T the result does not split a color of AS_1 or AS_2.

Data: n, list of association schemes
Result: List of coherent configurations of order n with two fibers
for $i \leq j$, $i + j = n$ **do**
  for AS_1, AS_2 *association schemes of orders* i, j **do**
    find all CB such that C above is a coherent configuration;
    add C to list if not already in list (up to isomorphism);
  end
end

Algorithm 1: Enumeration of coherent configurations with two fibers

Note that by the biregularity condition, if $\gcd(i, j) = 1$, there is only one good set, the one containing the whole $K_{i \to j}$.

In the second step we construct all partitions of $K_{i \to j}$ from good sets, and filter the partitions for ones that produce a coherent configuration.

3.2 Good sets enumeration

The main function of the C program that enumerates the good sets is listed in Appendix A. This function recursively enumerates all biregular subsets of valencies s_1, s_2 of $K_{i \to j}$ (in the C program, $i =$ `ord[0]`, $j =$ `ord[1]`). In level `level` of the recursion the s_1 out neighbors of vertex `level` are selected and added to the partial graph `st`. On the i-th level, the whole graph is already selected and is then checked for coherency.

The number of candidate graphs that need to be checked for coherency is limited by $\binom{j}{s_1}^i$, but since after each element of $[1, k]$ may only be used s_2 times, the actual number is smaller.

The function counts the number of times each element is used (`ns`), and removes the fully used ones from the set of available out neighbors `a`.

Note that $s_1 \cdot i = s_2 \cdot j$, therefore for a given i and j pair, only a small possible set of values may be used.

3.3 Partitions from good sets

The GAP function listed in Appendix B is the main part of the GAP program that implements the second step. This recursive function enumerates all partitions of `set` such that each cell is in `sets`.

Two limitations on the partitions are:

1. Only one representative of each orbit of the action of `group` on the partitions is required.

2. The function `compare` tells whether two sets are allowed to be in the same partition together (compatible).

The function works recursively by selecting element `e` of `set`, taking one representative `s2` containing `e` of each orbit of `group` on `sets`, and calculating all partitions of the set difference `set` $\setminus$ `s2`, from sets which are disjoint from `s2` and compatible with it. Instead of `group`, we are left with the set stabilizer in `group` of `s2`.

The function `compare` in this case checks that multiplying each set by any color of AS1 or AS2 does not split the other set.

The function `compute` is an optimization that speeds up `compare` by pre-computing some products. The extra parameter, `param` contains AS1 and AS2, in a form that, again, speeds up the calculations of `compare`.

The `group` passed in the initial invocation of this function is the direct product $Aut(\mathrm{AS}_1) \times Aut(\mathrm{AS}_2)$.

3.4 Coherent configurations with more than two fibers

To enumerate all coherent configurations of order n with $k+1$ fibers, $k \geq 2$, we start with a list of all coherent configurations of order less than n and exactly k fibers, and add to each of them a fiber of the needed size to complement the order to n.

$$\left(\begin{array}{c|c|c|c} \mathrm{AS}_1 & * & * & CB_1 \\ \hline * & \ddots & * & \vdots \\ \hline * & * & \mathrm{AS}_k & CB_k \\ \hline & & & \mathrm{AS}_{k+1} \end{array}\right)$$

If we start with a coherent configuration corresponding to the first k by k blocks in the above matrix, and try to extend it by AS_{k+1}, we only need to find the CB_i. It is not necessary to follow the steps described above for CB_i, since we know that

$$C_i = \left(\begin{array}{c|c} \mathrm{AS}_i & CB_i \\ \hline {CB_i}^T & \mathrm{AS}_{k+1} \end{array}\right)$$

is a coherent configuration with two fibers, so it is isomorphic to one in our already calculated list. For such coherent configuration, the group of isomorphisms that fix AS_i and AS_{k+1} is $CAut(\mathrm{AS}_i) \times CAut(\mathrm{AS}_{k+1})$.

4 Further discussion

4.1 Further optimization of the program

Some optimizations in the programs were not discussed above.

When looking for good sets, for every biregular graph, its complement is also tested, so we only need to work with $s1 \leq \frac{j}{2}$. If $s1 = \frac{j}{2}$, then the neighbors of 1 are taken out of $[1, j-1]$, instead of $[1, j]$.

The coherency test is independent for AS_1 and AS_2. In fact we calculate sets that are good for AS_1 and any other association scheme of order j, as well as sets that are good for AS_2 and any other association scheme of order i. Those that are good for AS_1 and AS_2, are exactly the intersection of those two sets of good sets.

In both construction of partitions, and search for coherent configurations with more than two fibers, we define a linear order on the association schemes (order of appearance in Hanaki and Miyamoto list), and make sure that in any coherent configuration, if $i < j$ then $\mathrm{AS}_i \leq \mathrm{AS}_j$, thus reducing repetitions.

The current code can enumerate all coherent configurations of order up to 15. For larger orders, further optimizations are required.

Potential optimizations are:

In the search for good sets, it is possible that some partial graphs are so far from good that they cannot be completed to good sets. Testing for such conditions at upper nodes may reduce the number of leaves considerably. Carrying the information currently in variables `ns` and `a` in the nodes may reduce the amount of calculations per node.

The `checkcoherent` function may probably be further optimized as well. While coherency of a set is not equivalent to coherency of its complement, the calculating both together may be faster than calculating each alone, as is done now.

In the construction of partitions, the function `compare` can be optimized to work faster. It may also disqualify more sets, by comparing not only the two sets in question, but also the already selected sets for the partition.

#	Rank	|Aut|	Aut		orbits	|CAut|	|AAut|
25	4	882	E49 : (C3 x S3)	26	$14^2, 2$	882	2
26	4	441	(C7 : C3) x (C7 : C3)	52	$7^4, 1^2$	882	2
27	4	40320	S8	4	30	40320	1
28	4	1152	((((E16:C3):C2):C3):C2):C2	14	$6, 24$	1152	1
29	4	192	((E8 : E4) : C3) : C2	28	$2, 12, 16$	192	1
30	4	168	E8 : (C7 : C3)	28	$1, 7, 8, 14$	168	1
106	7	360	(C15 : C4) x S3	7	30	720	2
107	7	72	(C6 x S3) : C2	27	$6, 24$	144	2
122	8	120	C2 x A5	12	30	720	6
123	8	120	S5	12	30	720	6

Table 4. Information on some association schemes of order 30

The initial group given to the partition construction function may also be based on $CAut$, instead of Aut.

4.2 Towards explanations of the results

Using the nomenclature described in [7] we aim for an explanation of the results presented in Table 2.

The association schemes of order 16 are mainly related to fusions of WFDF coherent configurations of order 16 (compare with similar objects of order 28 in [5]).

Some association schemes of order 16 (as well as of order 25) are amorphic schemes (see [6]).

The ASs of orders 19 and 23, as well as the rank 3 antisymmetric ASs of rank 27, are generated by doubly regular tournaments. The smallest DRT was already mentioned, as the smallest non-Schurian AS. See also the rapid increase in number of those DRTs as order increases.

The ASs of orders 26 and 29, as well as the rank 3 ASs of order 28, correspond to strongly regular graphs. For order 28 those are the classic Chang graphs.

The challenge of explaining the ASs of order 24, as well as those of rank larger than 3 with order 27 and 28, we leave for the future.

We now look into the ASs of rank 30. See Table 4 for some information about those association schemes. The numbers in the fifth column of the table are the Rank (number of orbitals) of the relevant automorphism group.

- $AS.30.25$ and $AS.30.26$ are wreath products of a scheme of order 2 and the DRT of order 15.
- The ASs numbers 27,28,29,30 are generated by symmetric BIBDs with parameters $v = 15$, $k = 7$, $\lambda = 3$. There are five such designs, three of them are self-dual, and a pair of dual designs.
- For the remaining two pairs of algebraically isomorphic schemes, we provide the calculated information in the table, and leave explanation and interpretation to the future.

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Supplements

A C function enumerating good sets

```c
void goodsets_r(FILE *f0, FILE *f1, set *st, int s1, int s2, int level) {
    int i, r0, r1;
    set *c1;

    if(level==ord[0]) {
        int j;
        set comp[MAXCOLORS];
        comp[0]=ord[0];
        r0=(f0!=NULL);
        r1=(f1!=NULL);
        checkcoherent(st, &r0, &r1);
        if(r0) {
            fprintf(f0,"[");
            gapsetsf(f0,st);
            fprintf(f0,"],\n");
        }
        if(r1) {
            fprintf(f1,"[");
            gapsetsf(f1,st);
            fprintf(f1,"],\n");
        }
        for(j=1;j<=ord[0];j++)
            comp[j]=DIFFERENCE(NBITS(ord[1]),st[j]);
        r0=(f0!=NULL);
        r1=(f1!=NULL);
        checkcoherent(comp, &r0, &r1);
        if(r0) {
            fprintf(f0,"[");
            gapsetsf(f0,comp);
            fprintf(f0,"],\n");
        }
        if(r1) {
            fprintf(f1,"[");
            gapsetsf(f1,comp);
            fprintf(f1,"],\n");
        }
        return;
    }

    if(level==0) {
        if(s1+s1==ord[1]) {
            c1=Combinations(NBITS(ord[1]-1),s1);
        } else {
            c1=Combinations(NBITS(ord[1]),s1);
        }
```

```
    } else {
        int k,m;
        int ns[BITS];
        set a;
        for(k=0;k<ord[1];k++) ns[k]=0;
        for(k=1;k<=level;k++) for(m=0;m<ord[1];m++)
            if (IS_IN(st[k],m))ns[m]++;
        a=NBITS(ord[1]);
        for(k=0;k<ord[1];k++)
            if(ns[k]==s2) a=DIFFERENCE(a,BITN(k));
        c1=Combinations(a,s1);
    }

    SSET_SETSIZE(st,level+1);
    for(i=1;i<=SSET_SIZE(c1);i++) {
        st[level+1]=c1[i];
        goodsets_r(f0, f1, st, s1, s2, level+1);
    }
    free(c1);
    return;
}
```

B GAP function enumerating partitions

```
PartitionsFromSetsAC := function(group, action, set, sets,
    compare, compute, param)
    local s1,s2,n1,p1,p2,l, orbs,e,i,nset, nsets;
    if set=[] then return [[]]; fi ;
    if sets=[] then return [] ; fi ;
    if not IsSubset(Union(sets),set) then return []; fi ;
    orbs:=Orbits(group,sets,action);
    e:=set[1];
    s1:=Filtered(orbs,x->e in Union(x));
    p1:=[];;
    for i in s1 do
      s2:=First(i,x->e in x);
      nset:=Difference(set, s2);
      l:=compute(s2,[],param);
      nsets:=Filtered(sets, x -> IsSubset(nset,x) and compare(s2
          ,x,param,l));
      Add(p1,List( PartitionsFromSetsAC(Stabilizer(group,s2,
          action), action,nset,nsets,compare,compute,param), x
          -> Union(x,[s2])));
    od ;

    return Union(p1);
end;
```

C GAP function enumerating coherent configurations with more than 2 fibers

```
CCs_morefibers3_1 := function(cc2, cc2b, cc2d, bcc, n)
#cc2[i] = list of CCs of order i with two fibers
# without fibers of size 1, smaller fiber first,
# for two fibers of the same size, the highest AS index is first
    .
# cc2b = list of pair of AS indexes of CCs
# cc2d = for each pair in cc2b, all CCs with those ASs (not up
    to isomorphism).
# bcc[i] = CCs of order i to add a fiber to.
# n = order of requested CCs.
  local ccs,ccg,cg,s,t,ii,c_c,a1,l,r1,r2,ast,nf,p1,nc, a2, k, jj
      , p3,lm,wm;
  ccs:=[];
  ccg:=[];
  for ii in [2..n-2] do
    r1:=[1..Size(bcc[ii])];
    for c_c in r1 do
      ast:=CC_ASFiberType(bcc[ii][c_c]);
      nf:=Size(ast);
      if n-ii>=ast[nf][1] then
        if n-ii=ast[nf][1] then
          r2:=[1..ast[nf][2]];
        else
          r2:=[1..Size(as[n-ii])];
        fi;
        for a1 in r2 do
          nc:=IdentityMat(n);
          nc{[1..ii]}{[1..ii]}:=bcc[ii][c_c]{[1..ii]}{[1..ii]};
          nc{[ii+1..n]}{[ii+1..n]}:=as[n-ii][a1]+100;
          lm:=[[nc]];
          k:=1;
          for t in [1..nf] do
            Add(lm,[]);
            p1:=Position(cc2b,[ast[t],[n-ii,a1]]);
            if p1=fail then
              break;
            fi;
            for a2 in cc2d[p1] do
              wm:=NullMat(n,n);
              jj:=ast[t][1];
              wm{[k..k+jj-1]}{[ii+1..n]}:=a2{[1..jj]}{[jj+1..
                  Size(a2)]}+200*(k+1);
              wm{[ii+1..n]}{[k..k+jj-1]}:=a2{[jj+1..Size(a2)
                  ]}{[1..jj]}+200*(k+2);
              Add(lm[Size(lm)],wm);
            od;
            k:=k+jj;
          od;
```

```
            for nc in List(Cartesian(lm),Sum) do
              if Size(Union(nc))>=Size(Union(nc^2)) and Size(Union
                  (nc))>=Size(Union(nc^3)) then
                p3:=FromColorMatrix(nc);
                if IsAS(p3) then
                  cg:=CAut_Graph(nc);
                  if ForAll(ccg,x->not BlissIsIsomorphicGraph(x,cg
                      )) then
                    Add(ccs, NormalizeColorMatrix(nc));
                    Add(ccg, cg);
                  fi;
                fi;
              fi;
            od;
          od;
        fi;
      od;
    od;
    return ccs;
end;
```

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