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ON ALMOST COMPLEX STRUCTURES ON FIBRE BUNDLES

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Abstract. If α is an almost complex structure on a manifold M then there is not a connection on M induced by α. In this paper the problem of connections on a fibre bundle $\pi : Y \to M$, $\dim M = \dim$ of fibres, which can be constructed from a given almost complex structure α on M only is explored.

Introduction

Let α be an almost complex structure (ACS) on a manifold M, $\dim M = 2m$, α is a (1,1)-tensor field on M such that $\alpha^2 = -Id_{TM}$. It is known, see [3], [4], that there is no connection on M, linear connection on the tangent bundle $p_M : TM \to M$, which is canonically induced by α. If α is an ACS on a fibre bundle $\pi : Y \to M$, $\dim M$ is the dimension of fibres, then the question of connections on Y entirely determined by α arises. Examples of such fibre bundles are $p_M : TM \to M$ and the cotangent bundle $\pi : T^*M \to M$. In this paper we construct connections from the given (1,1)-tensor field α on Y with emphasis on the ACS-case. If $Y = TM$ then there are some special geometric objects on TM which are in interesting relations to our topic. We have discussed them in [2]. In this paper all maps and manifolds are supossed to be smooth.

Connections and almost complex structures on Y

Let (x^i, y^i) be a local fibre chart on a fibre bundle $\pi : Y \to M$, $\dim M$ is the dimension of fibres.

Let us recall that a connection Γ on TY can be considered as a (1,1)-tensor field h_Γ (horizontal form of Γ), such that $T\pi h_\Gamma = T\pi, h_\Gamma(VY) = 0$, where $T\pi$ is the tangent map of the map π and VY is the vector bundle of all vertical vectors on Y, $h_\Gamma = dx^i \otimes \partial/\partial x^i + \Gamma^i_j(x,y)dx^j \otimes \partial/\partial y^i$. Then $h_\Gamma(TY) = H\Gamma$ is the so-called horizontal subbundle of Γ; $(x^i, y^i, dx^i, dy^i) \in H\Gamma$ if and only if $dy^i = \Gamma^i j dx^j, \Gamma^i_j(x,y)$ are said to be the functions of Γ.

Let $\alpha = (a^i_j(x,y)dx^j + b^i_j(x,y)dy^j) \otimes \partial/\partial x^i + (c^i_j(x,y)dx^j + h^i_j(x,y)dy^j) \otimes \partial/\partial y^i$ be a (1,1)-tensor field on Y. It is called vertical if $\alpha(VY) \subset VY$.

Denote $B : T\pi\alpha|_{VY} = b^i_j dy^j \otimes \partial/\partial x^i$.
It means that B can be considered as a vector bundle morphism $VY \to TM$ over π or $VY \to Yx_M TM$ over $Id|_Y$, i.e. as a section $Y \to V^*Y \otimes_Y TM$.

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Lemma 1. *A (1,1)-tensor field α is vertical iff $B = 0$.*

Proof is evident from the local form of α and B.

Remark 1. If B is regular, i.e. if it is an isomorphism, then we get the inverse vector bundle isomorphism $B^{-1} : Yx_M TM \to VY$ over $Id|_Y$, i.e. a section $B^{-1} : Y \to T^*M \otimes \otimes_Y VY, B^{-1} = \tilde{b}^i_j dx^j \otimes \partial/\partial y_i, \tilde{b}^i_k b^k_j = \delta^i_j$, i.e. a semibasic (1,1)-vector form with values in VY.

We will consider two cases.

1. $B \neq 0$, i.e. $\alpha(VY) \not\subset VY$, i.e. α is not vertical.
Let $\Gamma, dy^i = \Gamma^i_j dx^j$, be a connection on Y. Let $X = \eta^i \partial/\partial y^i$ be an arbitrary vertical vector on Y. Then $\alpha(X) = b^i_j \eta^j \partial/\partial x^i + h^i_j \eta^j \partial/\partial y^i$ is Γ-horizontal, i.e. $\alpha(X) \in H\Gamma$, if and only if $\Gamma^i_k b^k_j \eta^j = h^i_j \eta^j$. It means that $\alpha(VY) \subset H\Gamma$ iff

$$\Gamma^i_k b^k_j = h^i_j. \tag{1}$$

It immediately gives

Proposition 1. If and only if B is regular there is a unique connection Γ^2_α on Y such that $H\Gamma^2_\alpha = \alpha(VY)$.

The relation (1) induces that if B is regular then the functions of the connection Γ^2_α are $\Gamma^i_j = h^i_k \tilde{b}^k_j$.

We will construct another connections on Y when B is regular. Let $X = \xi^i \partial/\partial x^i + \eta^i \partial/\partial y^i$ be a vector on Y. Then $\alpha(X) = (a^i_j \xi^j + b^i_j \eta^j)\partial/\partial x^i + (c^i_j \xi^j + h^i_j \eta^j)\partial/\partial y^i$ is vertical if and only if

$$a^i_j \xi^j + b^i_j \eta^j = 0. \tag{2}$$

This leads

Proposition 2. If and only if B is regular there is a unique connection Γ^1_α on Y such that $\alpha(H\Gamma^1_\alpha) = VY$, i.e. with the functions $\Gamma^i_j = -\tilde{b}^i_k a^k_j$.

Remark 2. Recall that if φ is a semibasic (1,1)-form on Y with values in VY, i.e. if φ is a section $Y \to T^*M \otimes_Y VY$ and h_Γ is the horizontal form of a connection Γ on Y then $h_\Gamma + \varphi$ is the other connection on Y. So if B is regular then $h_{\Gamma^1_\alpha} + cB^{-1}$ and $h_{\Gamma^2_\alpha} + cB^{-1}, c \in \mathbb{R}$, are another connections on Y.

The (1,1)-tensor form α is a vector bundle morphism $TY \to TY$ over $Id|_{TM}$. Then

$$\alpha^2 = \alpha\alpha = [(a^i_s a^s_j + b^i_s c^s_j)dx^j + (a^i_s b^s_j + b^i_s h^s_j)dy^j] \otimes \partial/\partial x^i + \\ + [(c^i_s a^s_j + h^i_s c^s_j)dx^j + (c^i_s b^s_j + h^i_s h^s_j)dy^j] \otimes \partial/\partial y^i.$$

So α is an ACS on Y, i.e. $\alpha^2 = -Id|_{TY}$, iff

$$a^i_s a^s_j + b^i_s c^s_j = -\delta^i_j, \quad a^i_s b^s_j + b^i_s h^s_j = 0, \quad c^i_s a^s_j + h^i_s c^s_j = 0, \quad c^i_s b^s_j + h^i_s h^s_j = -\delta^i_j. \tag{3}$$

It is easy to see that if B is regular then the third and fourth equations of the relations (3) are the consequence of the first and second ones.

Proposition 3. Let α be a (1,1)-tensor field on Y such that B is regular. Then $\Gamma^1_\alpha = \Gamma^2_\alpha$ if and only if α^2 is vertical.

Proof. α^2 is vertical iff the second equation of (3) is satisfied, i.e. iff $a^i_j = b^i_s h^s_k \tilde{b}^k_j$. Then ${}^1\Gamma^i_j = -\tilde{b}^i_s a^s_j = h^i_k \tilde{b}^k_j = {}^2\Gamma^i_j$. Conversally, if $\Gamma^1_\alpha = \Gamma^2_\alpha$ then $-\tilde{b}^i_s a^s_j = h^i_s \tilde{b}^s_j$, i.e. $-a^i_s b^s_j = b^i_s h^s_j$, i.e. α^2 is vertical.

We will focus ourselves to the connections Γ which are invariant according to α, i.e. $\alpha(H\Gamma) \subset H\Gamma$.

Let $h_\Gamma = dx^i \otimes \partial/\partial x^i + \Gamma^i_j dx^j \otimes \partial/\partial y^i$ be an arbitrary connection on Y. Then

$$\alpha h_\Gamma = (a^i_j + b^i_k \Gamma^k_j) dx^j \otimes \partial/\partial x^i + (c^i_j + h^i_k \Gamma^k_j) dx^j \otimes \partial/\partial y^i.$$

Let $\overline{\Gamma}$ be another connection given by the equation $dy^i = \overline{\Gamma}^i_j dx^j$. Then $\alpha(H\Gamma) \subset H\overline{\Gamma}$ if and only if

$$\overline{\Gamma}^i_k (a^k_j + b^k_u \Gamma^u_j) = c^i_j + h^i_k \Gamma^k_j \quad \text{or} \tag{4}$$

$$c^i_j = \Gamma^i_k a^k_j - h^i_k \Gamma^k_j + \Gamma^i_k b^k_u \Gamma^u_j \quad \text{for } \overline{\Gamma} = \Gamma. \tag{5}$$

Consider the space $VY \otimes_Y T^*M$ of all semibasic VY-valued (1,1)-forms on Y. Let $\gamma = \gamma^i_j dx^j \otimes \partial/\partial y^i \in T^*M \otimes_Y VY$. Denote

$$\alpha^- : \gamma \to \alpha\gamma = b^i_t \gamma^t_j dx^j \otimes \partial/\partial x^i + h^i_t \gamma^t_j dx^j \otimes \partial/\partial y^i,\ T^*M \otimes_Y VY \to T^*M \otimes TY,$$

$$\alpha^+ : \gamma \to \gamma\alpha = (\gamma^i_k a^k_j dx^j + \gamma^i_k b^k_j dy^j) \otimes \partial/\partial y^i,\ T^*M \otimes_Y VY \to T^*Y \otimes VY.$$

Note that if B is regular then $\alpha^-(B^{-1}) = h_{\Gamma^2_\alpha}$ and $\alpha^+(B^{-1})$ is the vertical form $v_\Gamma = Id_{TY} - h_{\Gamma^1_\alpha}$ of the connection Γ^1_α.

Definition 1. Two (1,1)-tensor fields α_1, α_2 on Y will be called $(+,-)$-equivalent if $\alpha_1^- = \alpha_2^-$, $\alpha_1^+ = \alpha_2^+$.

It is evident that the relations ${}^1a^i_j = {}^2a^i_j$, ${}^1b^i_j = {}^2b^i_j$, ${}^1h^i_j = {}^2h^i_j$ are the coordinate conditions for α_1, α_2 to be $(+,-)$-equivalent.

The relation (4) immediately yields

Proposition 4. Let $\Gamma, \overline{\Gamma}$ be connections on Y. Then in every class of the $(+,-)$-equivalent (1,1)-tensor fields on Y there exists a unique (1,1)-tensor field $\alpha_{\Gamma,\overline{\Gamma}}$ such that $\alpha_{\Gamma,\overline{\Gamma}}(H\Gamma) \subset H\overline{\Gamma}$.

If $\Gamma = \overline{\Gamma}$ then we use the denotation α_Γ instead of $\alpha_{\Gamma,\Gamma}$.

Proposition 5. Let α be such a (1,1)-tensor field on Y that B is regular. Then $\alpha_{\Gamma^1_\alpha} = \alpha_{\Gamma^2_\alpha}$ and $\alpha_{\Gamma^1_\alpha}$ cannot be an almost complex structure on Y.

Proof. By the relation (5) in both cases of Γ^1_α and Γ^2_α we get $c^i_j = h^i_t \tilde{b}^t_s a^s_j$. So $\alpha_{\Gamma^1_\alpha} = \alpha_{\Gamma^2_\alpha} = (a^i_j dx^j + b^i_j dy^j) \otimes \partial/\partial x^i + (h^i_t \tilde{b}^t_s a^s_j dx^j + h^i_j dy^j) \otimes \partial/\partial y^i$.

If $\alpha_{\Gamma^1_\alpha}$ is an ACS then the first and second equations of (3) read

$$a^i_s a^s_j + b^i_s h^s_t \tilde{b}^t_k a^k_j = -\delta^i_j, \quad \tilde{b}^i_t a^t_j + h^i_t \tilde{b}^t_j = 0.$$

Then $a^i_s a^s_j - b^i_s \tilde{b}^s_t a^t_k a^k_j = -\delta^i_j$. It is not possible. So $\alpha_{\Gamma^1_\alpha}$ cannot be an almost complex structure on Y.

Definition 2. Let $(1,1)_B$ denote the set of all (1,1)-tensor field α on Y such that B is regular. We will say that two (1,1)-tensor field $\alpha_1, \alpha_2 \in (1,1)_B$ are (+)-equivalent if $\alpha_1^+ = \alpha_2^+$.

In coordinates, α_1 and α_2 are (+)-equivalent iff ${}^1a^i_j = {}^2a^i_j$, ${}^1b^i_j = {}^2b^i_j$, $\det {}^1b^i_j \neq 0$, $\det {}^2b^i_j \neq 0$.

Proposition 6. In every class of all (+)-equivalent (1,1)-tensor fields there is a unique almost complex structure on Y.

Proof in coordinates. A class of all (+)-equivalent (1,1)-tensor fields is given by the local functions $a^i_j, b^i_j, \det b^i_j \neq 0$. By the first and second equations of the relation (3) a tensor field of this class is an ACS iff $c^i_j = -\tilde{b}^i_j - \tilde{b}^i_k a^k_s a^s_j$, $h^i_j = -\tilde{b}^i_s a^s_k b^k_j$. It completes our proof.

Remark 3. The same can be said for the class of (−)-equivalent tensor fields.

Remark 4. If α is an ACS on Y then α is vertical and so $\Gamma^1_\alpha = \Gamma^2_\alpha$.

Proposition 7. Let Γ be a connection on Y. Let $B : Y \to V^*Y \otimes_Y TM$, $B = b^i_j dy^j \otimes \partial/\partial x^i$, be a vector bundle isomorphism $VY \to TM$ over π. Then there exists a unique almost complex structure α on Y such that $T\pi\alpha|_{VY} = B$ and $\Gamma^1_\alpha = \Gamma = \Gamma^2_\alpha$.

Proof. Let Γ^i_j be the functions of Γ. Let α be an arbitrary (1,1)-tensor field on Y such that $T\pi\alpha|_{VY} = B$ and $\Gamma^1_\alpha = \Gamma = \Gamma^2_\alpha$. Then $\Gamma^i_j = -\tilde{b}^i_k a^k_j$, $\Gamma^i_j = h^i_k \tilde{b}^k_j$, i.e. $a^i_j = -b^i_s \Gamma^s_j$, $h^i_j = \Gamma^i_s b^s_j$ and the second equation of (3) is satisfied. By the first equation of (3) α is an ACS iff $c^i_j = -\tilde{b}^i_j - \Gamma^t_s b^s_u \Gamma^u_j$. So such an ACS locally exists and is unique.

We are turning to the second case of α.

2. Let $B = T\pi\alpha|_{VY} = 0$, i.e. $\alpha(VY) \subset VY$. We have

$$\begin{aligned}
\alpha &= a^i_j dx^j \otimes \partial/\partial x^i + (c^i_j dx^j + h^i_j dy^j) \otimes \partial/\partial y^i,\\
A &:= T\pi\alpha = a^i_j dx^j \otimes \partial/\partial x^i\\
H &:= \alpha|_{VY} = h^i_j dy^j \otimes \partial/\partial y^i.
\end{aligned}$$

So A is a section $Y \to T^*M \otimes_Y TM$ determining a vector bundle morphism $TY \to TM$ over $\pi : Y \to M$ or $Yx_M TM \to Yx_M TM$ over Id_Y and H is a section $Y \to V^*Y \otimes VY$ determining a vector bundle morphism $VY \to VY$ over Id_Y.

Let $\Gamma, \overline{\Gamma}$ be two connections on Y with the local functions $\Gamma^i_j, \overline{\Gamma}^i_j$. When $B = 0$ the equations (4) and (5) read

$$c^i_j = \overline{\Gamma}^i_k a^k_j - h^i_k \Gamma^k_j, \tag{4'}$$

$$c^i_j = \Gamma^i_k a^k_j - h^i_k \Gamma^k_j. \tag{5'}$$

Proposition 4 can be reformulated as follows

Proposition 8. Let $H : Y \to V^*Y \otimes VY,\ A : Y \to T^*M \otimes_Y TM$ be two sections. Let $\Gamma, \overline{\Gamma}$ be two connections on Y. Then there is a unique vertical (1,1)-tensor field $\alpha(A, H, \Gamma, \overline{\Gamma})$ on Y such that $\alpha|_{VY} = H,\ T\pi\alpha = A,\ \alpha(H\Gamma) \subset H\overline{\Gamma}$.

If $\Gamma = \overline{\Gamma}$ we use the denotation $\alpha(A, H, \Gamma)$ instead of $\alpha(A, H, \Gamma, \overline{\Gamma})$.

In the case of a vertical (1,1)-tensor field α the coordinate conditions (3) for α to be an ACS are of the form

$$(3') \qquad a^i_s a^s_j = -\delta^i_j, \quad c^i_s a^s_j + h^i_s c^s_j = 0, \quad h^i_s h^s_h = -\delta^i_j.$$

Preserving the above denotations we have the following vector bundle morphism on $T^*M \otimes_Y VY$ over Id_Y:

$$\begin{aligned}
&H^- : \gamma \to H\gamma = h^i_k \gamma^k_j dx^j \otimes \partial/\partial y^i,\ \gamma \in T^*M \otimes_Y VY, \text{ so } H^- = \alpha^-,\\
&A^+ : \gamma \to \gamma A = \gamma^i_k a^k_j dx^j \otimes \partial/\partial y^i, \text{ so } A^+ = \alpha^+,\\
&\mathcal{H} : A^+ - H^- : \gamma \to (\gamma^i_k a^k_j - h^i_k \gamma^k_j) dx^j \otimes \partial/\partial y^i,\\
&\overline{\mathcal{H}} := A^+ + H^- : \gamma \to (\gamma^i_k a^k_j + h^i_k \gamma^k_j) dx^j \otimes \partial/\partial y^i.
\end{aligned}$$

The relation (5') immediately gives

Proposition 9. Let α be a such vertical (1,1)-tensor field on Y that the map $\mathcal{H}$ is a vector bundle isomorphism on $T^*M \otimes_Y VY$ over Id_Y. Then there is a unique connection Γ on Y such that $\alpha(H\Gamma) \subset H\Gamma$.

Lemma 2. *If a vertical (1,1)-tensor field α is an almost complex structure on Y then the maps $\mathcal{H}$ and $\overline{\mathcal{H}}$ are not isomorphisms on $T^*M \otimes_Y VY$.*

Proof. The map $(H^- + A^+)\mathcal{H} = H^-\mathcal{H} + A^+\mathcal{H} : \gamma \to (h^i_t \gamma^t_k a^k_j - h^i_t h^t_k \gamma^k_j) + (\gamma^i_t a^t_k a^k_j - -h^i_t \gamma^t_k a^k_j)$ is a vector bundle morphism on $T^*Y \otimes_Y VY$. If α is on ACS on Y then by (3') we get $(H^- + A^+)\mathcal{H} = 0$. If $\mathcal{H}$ is regular then $H^- + A^+ = 0$. But the equation $H\gamma = -\gamma A$ is satisfied for all $\gamma \in T^*M \otimes_Y VY$ if and only if $H = k \cdot Id = -A,\ k \in \mathbb{R}$. Then $h^i_k h^k_j = k^2 \delta^i_j$, i.e. $k^2 = -1$. It is contrary with $k \in \mathbb{R}$. Analogously the supposition "$\overline{\mathcal{H}}$ is regular" leads to contradiction.

Remark 5. If α is a vertical ACS on Y then according to (5') such a connection Γ that $\alpha(H\Gamma) \subset H\Gamma$ can but not have to exist. If it exists then it does not need to be unique.

Remark 6. Let $A : Y \to T^*M \otimes_Y TM$ be an ACS on Yx_MTM. Let $H : Y \to V^*Y \otimes VY$ be an ACS on VY. In view of the relation (3') there exists a vertical ACS α on Y such that $T\pi\alpha = A,\ \alpha|_{VY} = H$ and is not unique.

Proposition 10. Let $A : Y \to T^*M \otimes_Y TM$ be an ACS on Yx_MTM. Let $H : Y \to \to V^*Y \otimes VY$ be an ACS on VY. Let Γ be a connection on Y. Then the vertical (1,1)-tensor field $\alpha(A, H, \Gamma)$ described in Proposition 8 is an almost complex structure.

Proof. By Proposition 8, $\alpha(A, H, \Gamma)$ is the unique vertical (1,1)-tensor field on Y such that $\alpha(A, H, \Gamma)|_{VY} = H,\ T\pi\alpha(A, H, \Gamma) = A$ and $\alpha(A, H, \Gamma)(H\Gamma) \subset H\Gamma$. If $A = a^i_j dx^j \otimes$

$\otimes\partial/\partial x^i$, $H = h^i_j dy^j \otimes \partial/\partial y^i$ and Γ^i_j are the functions of Γ then the coordinates c^i_j of $\alpha(A,H,\Gamma)$ are determined by (5'). So $\alpha(A,H,\Gamma) = a^i_j dx^j \otimes \partial/\partial x^i + [(\Gamma^i_k a^k_j - h^i_k \Gamma^k_j)dx^j + + h^i_j dy^j] \otimes \partial/\partial y^i$. The functions a^i_j, h^i_j satisfy the first and third equations of (3'). Then $c^i_s a^s_j + h^i_s c^s_j = (\Gamma^i_k a^k_s - h^i_k \Gamma^k_s)a^s_j + h^i_s(\Gamma^s_k a^k_j - h^s_k \Gamma^k_j) = 0$. So $\alpha(A,H,\Gamma)$ satisfies the relations 3 and is an almost complex structure.

Remark 7. Let $\pi : Y \to M$ be a vector fibre bundle. Let α be a VB-(1,1)-tensor field, i.e. $\alpha(X)$ is a linear projectable vector field on Y for all linear projectable vector fields X on Y. In a local fibre chart $\alpha = a^i_j(x)dx^j \otimes \partial/\partial x^i + [c^i_{jk}(x)y^k dx^j + h^i_j(x)dy^j] \otimes \otimes\partial/\partial y^i$, see [1]. In this case $\alpha|_{VY} = H$ is a vector bundle morphism on Y over Id_M with the coordinate expression: $\overline{x}^i = x^i$, $\overline{y}^i = h^i_j(x)y^j$. These equations with the added following ones $d\overline{x}^i = dx^i$, $d\overline{y}^i = h^i_{jk}y^j dx^k + h^i_j dy^j$, where we use $\frac{\partial f}{\partial x^i} := f_i$, determine the tangent map TH. Let Γ, $\Gamma^i_j(x,y) = \Gamma^i_{jk}y^j$, be a linear connection on Y. Then it is easy to deduce that the equations

$$\Gamma^i_{js}h^s_k - h^i_s\Gamma^s_{jk} = h^i_{kj}$$

are the coordinate conditions under which $TH(H\Gamma) \subset H\Gamma$. The solution Γ^i_{js} of these equations can but not has to exist. Let $\alpha(H\Gamma) \subset H\Gamma$. Then by (5'): $c^i_{kj} = \Gamma^i_{sj}a^s_k - -h^i_s\Gamma^s_{kj}$. If Γ is without torsion then the conditions $TH(H\Gamma) \subset H\Gamma$, $\alpha(H\Gamma) \subset H\Gamma$ lead to

$$\Gamma^i_{js}(h^s_k - a^s_k) = h^i_{kj} - c^i_{kj}. \tag{6}$$

If $H - A$ has sense (for instance in the case of $y = TM$) and if $H - A$ is regular then there is a unique solution Γ^i_{js} of (6). For example if $A = -H$, and H is regular then $\Gamma^i_{js} = \frac{1}{2}(h^i_{ks} - c^i_{ks})\tilde{h}^k_s$. In view of Proposition 10 we can say that if α is a symmetric VB-almost complex structure on TM such that $A = -H$, then there exists a unique symmetric linear connection such that $TH(H\Gamma) \subset H\Gamma$,
$\alpha(H\Gamma) \subset H\Gamma$. We will deal in detail with such an almost complex structure in our other paper.

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